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ABSTRACT

To test the theory that retardates are particularly slow in forming some visual habits, especially attention, a series of experiments were performed using a laboratory device which forced subjects to discriminate between color and form in exchange for an incentive reward. Stochastic models were applied to tests of lower level retardates which measured original learning reversals, effects of intelligence, stimulus factors, reward schedule, and transfer operations. It was found that retardates have difficulty attending to the relevant stimulus. Theoretical background, test procedures, results, and conclusions are described in detail. (EMH)

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AN ATTENTION THEORY OF RETARDATE DISCRIMINATION LEARNING

PROGRESS REPORT NO. 3
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Studies of the learning and transfer processes in mental
defectives and applications to problems of training

TR004 B5

Role of Attention in Retardate Discrimination Learning¹

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Synopsis

Retardates have been found particularly slow in forming some simple visual habits, even slower than expected from low mental age. (Stevenson & Iscoe, 1955; House & Zeaman, 1958a; Zeaman & House, 1960a; Girardeau, 1959). Reasons for the learning deficit, we believe, lie not in the area of instrumental learning, but rather that of attention. In support of this notion, data from a series of experiments with moderately retarded, trainable children have been collated here and organized by a theory of attention.

Method

Experimental

The experimental techniques we employ are derived from those of Harlow (1959). His Wisconsin General Test Apparatus has been adapted for use with retarded children, as it provides a flexible tool for the study of basic discriminative processes. Figure 1 depicts the major features of the apparatus:

Insert Fig. 1 about here

a tray displaying two stimulus objects (one of which is baited with candy), and a one-way-vision screen separating the subject and experimenter. The experimenter arranges the stimuli on the tray behind the screen, hides a candy under one, and slides the tray out to the subject for choice.

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We gratefully acknowledge the help of several collaborators. Herbert M. Kaufman developed a number of the equations for extensions of the basic model, in particular the ~~the~~ submodel. Computer programming was expertly done for us by Lester Hyman. Research contributions were made by USPHS Postdoctoral Fellows, Robert Orlando, Jerome Smith, and Bryan Shepp, and a number of others. Without the cooperation of the administration of the Mansfield Training School, Neil Dayton, John Cassell, and Louis Boly the research reported here would not have been possible.

This work was done in part at the M.I.T. Computation Center, Cambridge, Massachusetts.

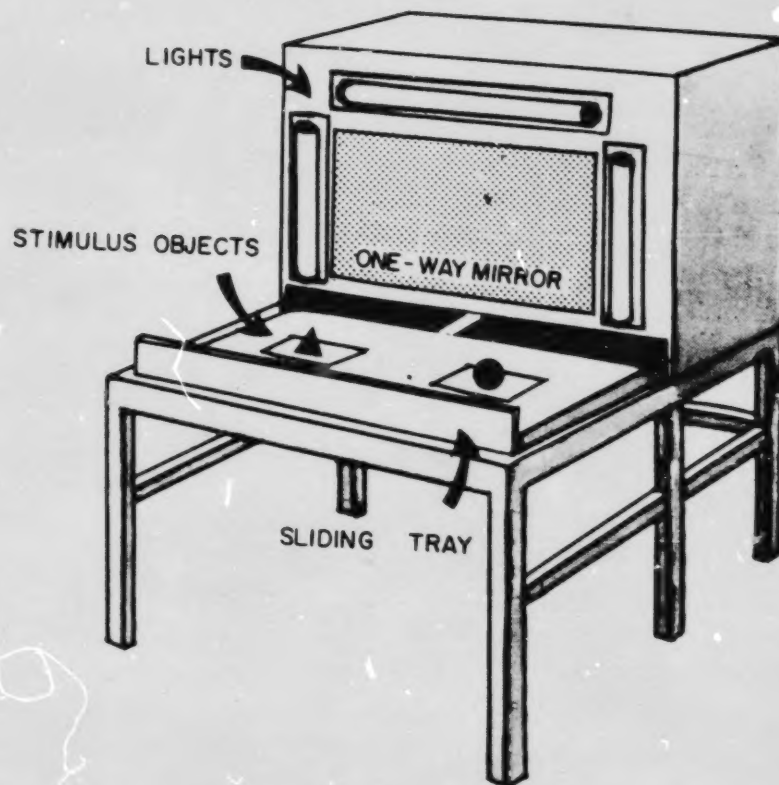


Figure 1. Discrimination learning apparatus.

This completes one trial of a two-choice simultaneous, visual discrimination learning problem. Procedure continues trialwise for original learning problems, reversals, schedules, changes of incentive, and a variety of transfer operations.

Analysis of Discrimination Learning Curves

Group curves. - Analyses of the forms of discrimination learning curves may provide some clues about the nature of underlying processes. Let us begin with a look at some data. For a group of 50 retarded children (MA: 2-6) who learned to discriminate between two objects differing in just color and form, Figure 2 presents a traditional group learning curve: a plot of trials

Insert Fig. 2 about here

against percentage correct responses of the group. The curve is gradually rising with overall negative acceleration. From this we might erroneously infer a single gradual underlying process similar to the growth of habit strength in Hull's system.

Homogeneous groups. - Such traditional functions, however, are likely to obscure characteristics of individual performance, a fact well demonstrated by Hayes (1953), Sidman (1952), and Estes (1960), who have shown that the average curve of a group does not necessarily have the same form as those of individual members of a heterogeneous group. Since individual discrimination learning functions are usually too irregular to permit accurate assessment of form, a common alternative procedure averages together performances of homogeneous groups.

Wide individual differences in rate of learning can be eliminated by grouping subjects who take the same number of days to achieve criterion. The 50 subjects described above achieved a learning criterion of 20 correct responses in a daily run of 25 trials on one of six days of training. Those learning on the first day have their performances averaged and plotted to the left in Figure 3. Their function, labelled Day 1, is a sharply rising and negatively

Insert Fig. 3 about here

accelerated curve. The Day 2 Group may be classified similarly, but as we move out to Days 3, 4, 5, and 6, a new form emerges. The learning curves become more and more ogival in shape. The slow learners appear to stay close to chance performance for varying numbers of trials, but once performance starts to improve it moves relatively fast.

Backward learning curves. - Furthermore, the final rates of all groups are quite similar. This is more easily seen in Figure 4 which moves all the functions in Figure 3 to the right so that the last point of every function

Insert Fig. 4 about here

has the same abscissa value, the next-to-last point of all functions has the adjacent abscissa value, and so on. Averaging together plots of this kind yields "backward learning curves," a procedure invented by Hayes (1953) to bring out the shapes of group curves in the vicinity of criterion. Statistically, these backward curves are not different in slope.

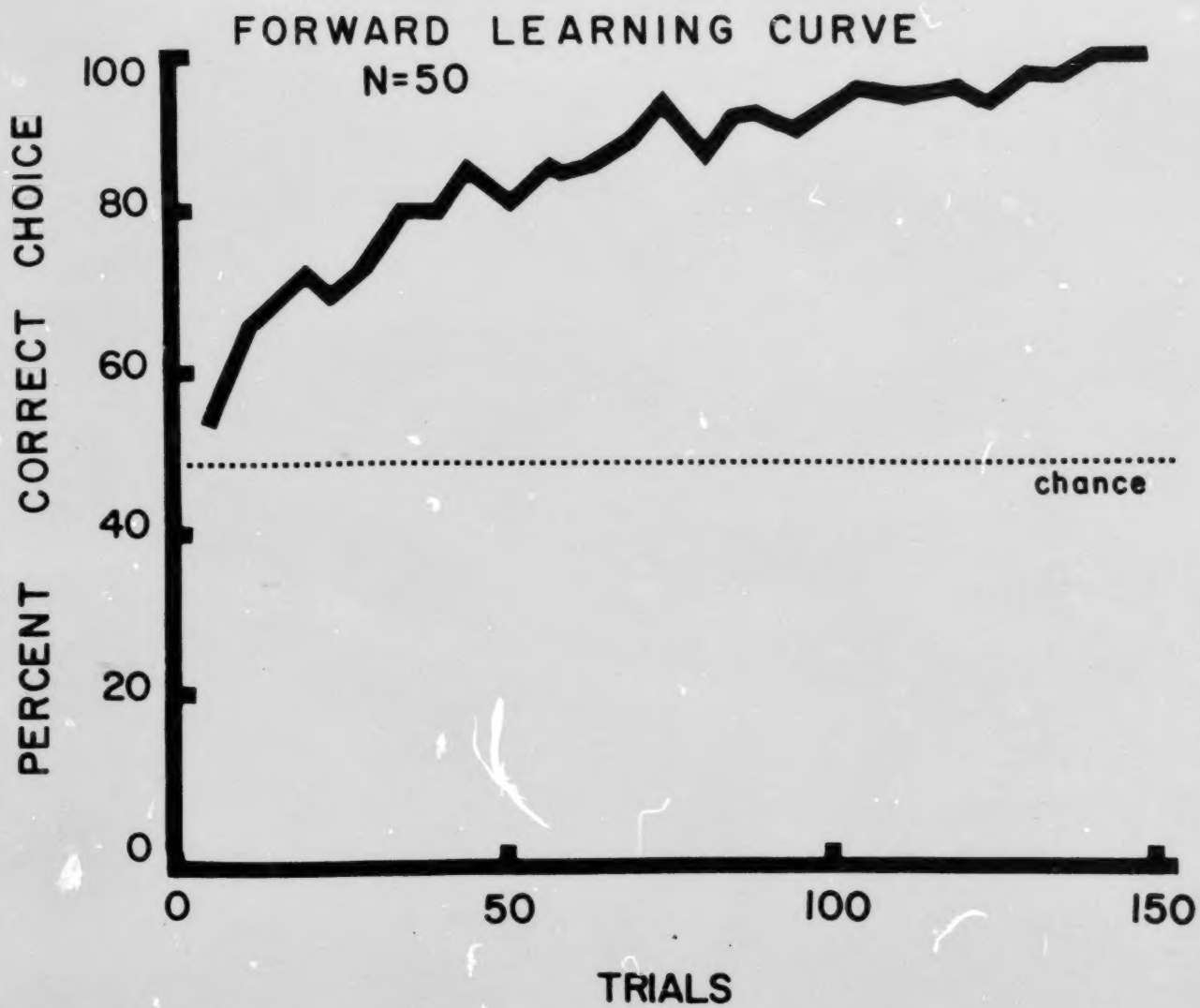


Figure 2. Average discrimination learning curve of a group of retardates.

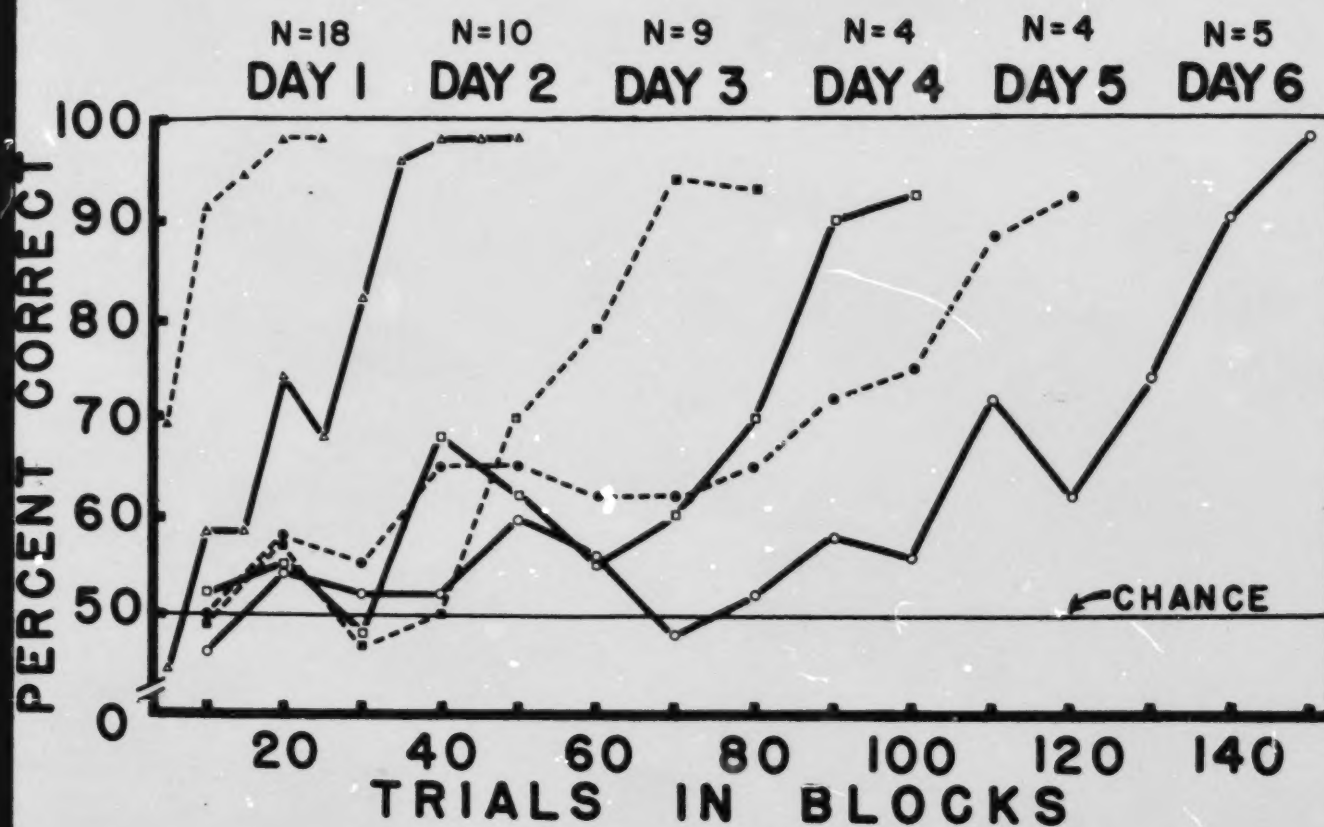


Figure 3. Forward learning curves of subgroups requiring various numbers of training days to reach criterion. The number of subjects in each group appears at top.

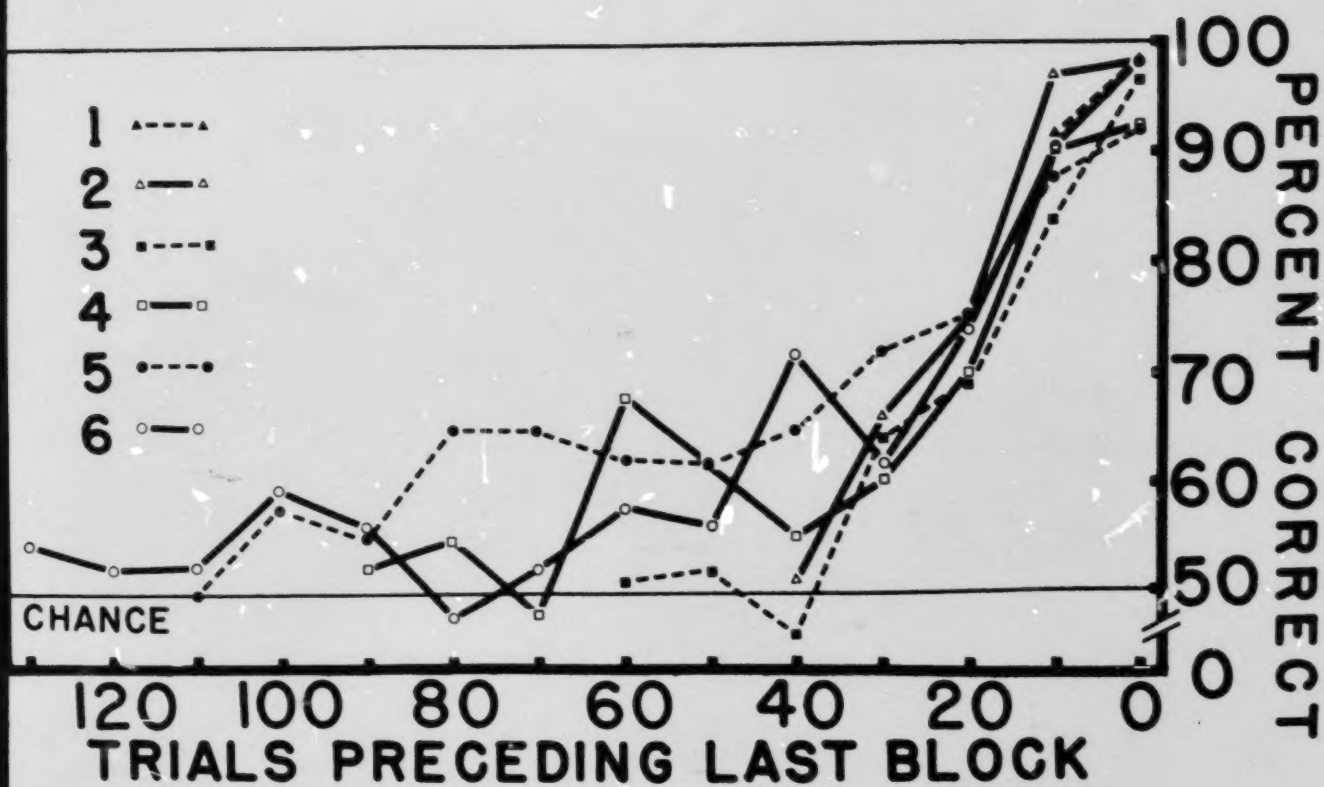


Figure 4. The functions of Figure 3 have been regrouped prior to averaging for a "Backward" learning curve. The contrast in form of backward and forward learning curves (Figure 1) is marked.

Our first clue about the nature of underlying processes comes, then, from Figures 3 and 4. The difference between fast and slow learning is not so much the rate at which improvement takes place, once it starts, but rather the number of trials for learning to start. At this juncture we anticipate our later arguments by introducing the surmise that the length of the initial, flat, chance-level stages of the performance curves is controlled primarily by an attention process, while the final, sharply rising portion of the curves is largely indicative of instrumental discriminative learning.

Effects of Intelligence

The relevance of this conjecture to the problem of retardation is established by showing that the length of the initial flat part of the ogive (the attention phase) varies with intelligence. Figure 5 presents backward

Insert Fig. 5 about here

learning curves for two groups of naive subjects differing in MA. The task is still a Color-Form Object Discrimination as before, but data from failing subjects have been included to indicate the relative difficulty of the task. Of the 31 subjects in the MA 4-6 year Group, 20 learned the discrimination with the median subject taking 2 days (50 trials). The backward learning curve of this group is given the abscissa locus of the median subject. At the lower MA level of 2-4 years, 21 of 40 subjects achieved criterion with a median score of 9 days. The backward learning curve of this group is shown at the right. Both groups of nonlearners have their average performances plotted for 10 days of practice.

To show that the performance difference between the higher and lower MA groups lies entirely in the length of the initial, chance plateau, the points of the MA 4-6 function have been replotted at the same abscissa locus as the MA 2-4 Group. The final rates of approach to criterion are the same.

The MA 2-4 Group (learners and nonlearners) had about the same average CA (140 months) as the MA 4-6 Group (147 months). The mean IQ of the MA 2-4 Group was 30, as compared to 41 for the higher MA Group.

The overall picture emerging from analyses of the shapes of discrimination learning curves is one of discontinuity, in the sense that performance is either improving rapidly or not at all. An apparent discontinuity of this sort could be produced by two continuous processes, with the rate of the second dependent upon the level of the first. The tentative identification of the process underlying the initial phase as attentive in nature leads us to look next at a general class of input variables that presumably exercise some control of attention - stimulus factors.

Stimulus Factors

Differences in number or kind of stimulus dimensions clearly affect acquisition of visual discriminations. Figure 6 presents backward learning

Insert Fig. 6 about here

curves of retardates, MA 4-6, for a variety of stimulus conditions. At the far left, a group of 8 naive subjects learned to discriminate multidimensional stimulus objects in a median of 25 trials (one day). Examples of multidimensionally different stimuli (junk) are an aluminum potcover versus a green plastic soap dish and a toy hat versus a tobacco can. None of the 8 naive subjects

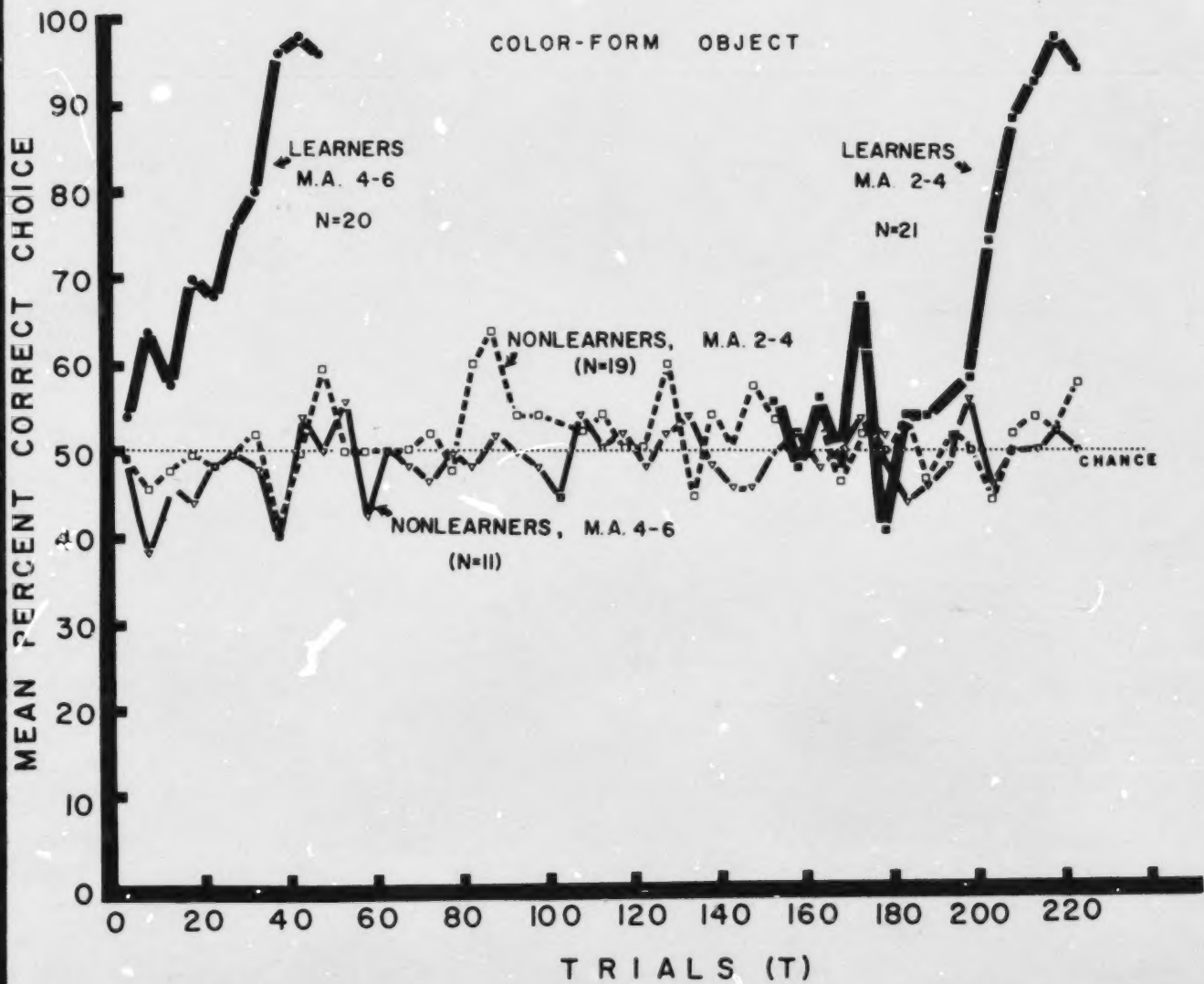


Figure 5. Effects of intelligence on discrimination learning are shown in the average performances of four groups classified by mental age and achievement. Backward curves are plotted for the two groups of learners.

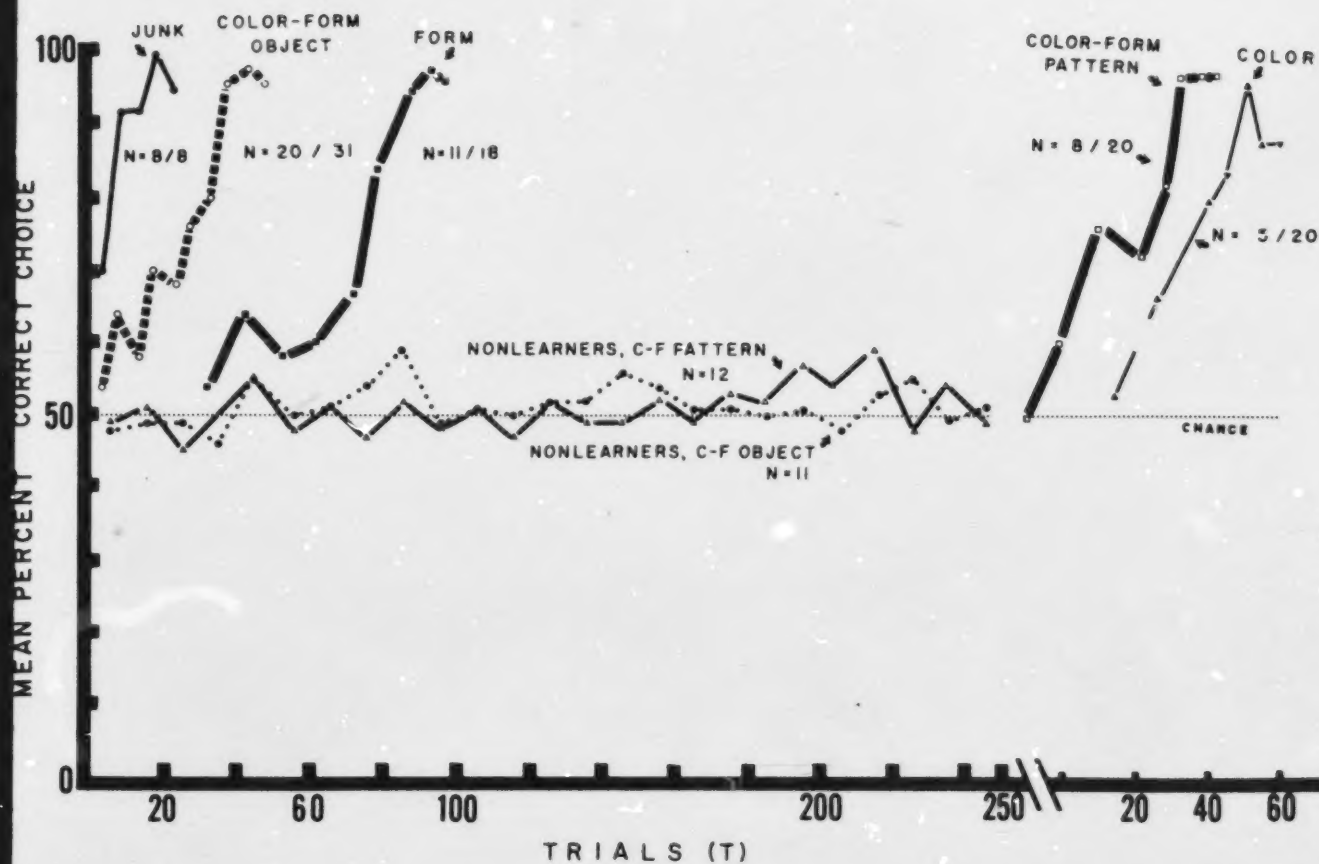


Figure 6. Backward learning curves for the discrimination of several kinds of stimuli. Average performance levels of nonlearners stay close to chance.

failed; none took more than three days to learn. A great many experienced subjects have been run on this problem and it is a rare subject who fails.

Junk discrimination is considerably easier than discrimination of objects differing in both color and form. The Color-Form Object functions from Figure 5 are recopied for comparison.

Having only form as a relevant aspect of two objects yields the middlemost backward learning curve of Figure 6. Of 18 subjects, 11 learned to distinguish pairs of objects such as a triangle versus a circle in a median number of 100 trials. These were experienced subjects who had previously been trained on an easier discrimination. For this reason, the difficulty of the Object-Form Problem, and also, the Object-Color Problem may be underestimated.

At the far right of Figure 6 is the backward learning curve of subjects given the task of learning a color discrimination. Two objects differing only in color, such as a red square versus a green square, are most difficult to discriminate. A minority of 5 of a total of 20 subjects learned this problem in the ten days allotted. Given that the median subject failed, abscissa locus for the backward learning curve is determinate only to the extent of falling to the right of 250 trials.

Somewhere in difficulty between the Object-Form and the Object-Color Problem comes the Pattern-Color-Form Discrimination. The three-dimensional objects used for the Color-Form Object Discrimination were presented as flat, two-dimensional patterns (e.g., red triangle vs. yellow cross). This seemingly minor change in the nature of the cues resulted in a large difference in performance. A minority of 8 out of 20 subjects learned to solve this problem, which calls for a placement of the backward learning curve in Figure 6 somewhere between 250 trials and the Color function.

Despite the wide differences in difficulty among these five problems the slopes of the backward learning curves do not vary widely, a fact to be claimed by the attention theory which follows. In summary, Figure 6 leads to the following inferences: (a) that the number of relevant stimulus dimensions controls speed of discrimination learning, supported by the rank ordering in difficulty of the Junk, Color-Form Object, Object Form (or Color) conditions; (b) that the kind of relevant dimension also exerts control, supported by the rank ordering of the Object-Form and Object-Color conditions, and the Object versus Pattern comparisons; and (c) that the control exerted by these factors is largely upon the duration of the initial flat portion of the discrimination performance curves. It is possible that what we are calling kind of cue may turn out to be number of cues. Three dimensional objects may present more differential aspects than two dimensional analogues, and form may be made up of more subdimensions than color. We have no evidence as yet on this point, and it is not crucial to the theory.

Details of method and procedure for the experiments cited above and later have been previously published (House and Zeaman, 1960a; House and Zeaman, 1962a), but a description of the theory we use to analyze and re-order the data has not been published. Up to this point our theoretical conjectures have been sketched only vaguely, so we turn now to the task of explicating more precisely our major assumptions.

Theory

Qualitative presentation

Several quantitative discrimination theories, e.g., those of Burke and Estes (1957), Bush and Mosteller (1951), Restle (1955) and Atkinson (1958), predict either negatively accelerated learning functions or ogival forms lacking the abrupt transitions seen in Figure 3 to 6. These all assume that

the subject samples relevant stimuli on every trial. The shapes of our discrimination functions suggest a departure from this assumption by allowing for the possibility that the relevant cues are not attended to on every trial. The subject may have to learn to attend to the relevant stimulus dimension.

Thus, instead of having only to learn the correct instrumental response, the subject must learn a chain of two responses: (a) attending to the relevant dimension, and (b) approaching the correct cue of that dimension.

Wyckoff (1952) has constructed an "observing response" model incorporating these notions which we will modify and extend to apply to our experimental situations. At the beginning of every trial, a variety of stimulus dimensions, both relevant and irrelevant, is presented. We assume the subject's attention is limited, so that not all stimuli are effective as cues. As a simplifying assumption, we postulate in our first submodel that the subject observes or attends to only one dimension at a time. A trial is concluded when the subject responds instrumentally with a choice of one of the two stimulus objects presented. If the correct object is selected, reward is given which acts to strengthen both the instrumental response and whichever observing response immediately preceded it. If the incorrect object is selected, reward is withheld. This weakens both responses in the immediately preceding chain, observing and instrumental.

A diagram may facilitate exposition. At the top of Figure 7 is shown a somewhat walled subject viewing a simultaneous presentation of two stimulus

Insert Fig. 7 about here

objects differing in color and form. At the moment of choice, the subject may be observing either the relevant stimulus dimension S (which in the diagram is form) or an irrelevant dimension S' (color or position, in the example). If the relevant observing response O_1 is made, the specific values or cues of the form dimension, triangle (s_1) and circle (s_1') are seen. If attention is focussed (O_2) on the irrelevant color dimension, the yellow and blue elements (s_2 and s_2') of this dimension become effective.

Immediately following O_1 the subject makes one of two instrumental responses, approaching the positive cue $-(R_1)$, or approaching the negative cue $-(R_1')$; following O_2 an approach is made to one of the cues of the irrelevant dimension. None of these responses is directly observable, neither the instrumental approach responses nor the observing responses. The only observable responses in the system are the overt picking up (R_c) of the correct (baited) stimulus object, or choice of the incorrect object (R_c'). The theory predicts the probabilities of these observable responses.

The bottom half of Figure 7 pictures the subject attending to the form dimension. The theory allows the possibility that only one of the specific cues (circle) of the relevant dimension might be seen, as would certainly be true if the stimulus objects were presented successively rather than simultaneously. In this event, the subject can then either approach the circle (R_1), or avoid it (R_1'). At present, for the two-choice experiment, no distinction is made between approaching the circle or avoiding the triangle.

The diagram portrays the irrelevant color and position cues as inconsistently related to the candy reinforcement. For the simpler models of the theory, only a single dimension is relevant and all others are randomly associated with reward. For more complex models the number of relevant dimensions and the degree of relevance are made parameters of the system.

The set of all dimensions both relevant and irrelevant, present at the moment before choice, is designated S^* in Figure 7. The competing observing responses O_1 and O_2 are elicited by S^* or some subset of S^* . Appropriate transfer experiments could be designed to reveal which aspects of S^* elicit the observing responses. For the present, we simply assume that whatever elicits O_1 is present on every trial.

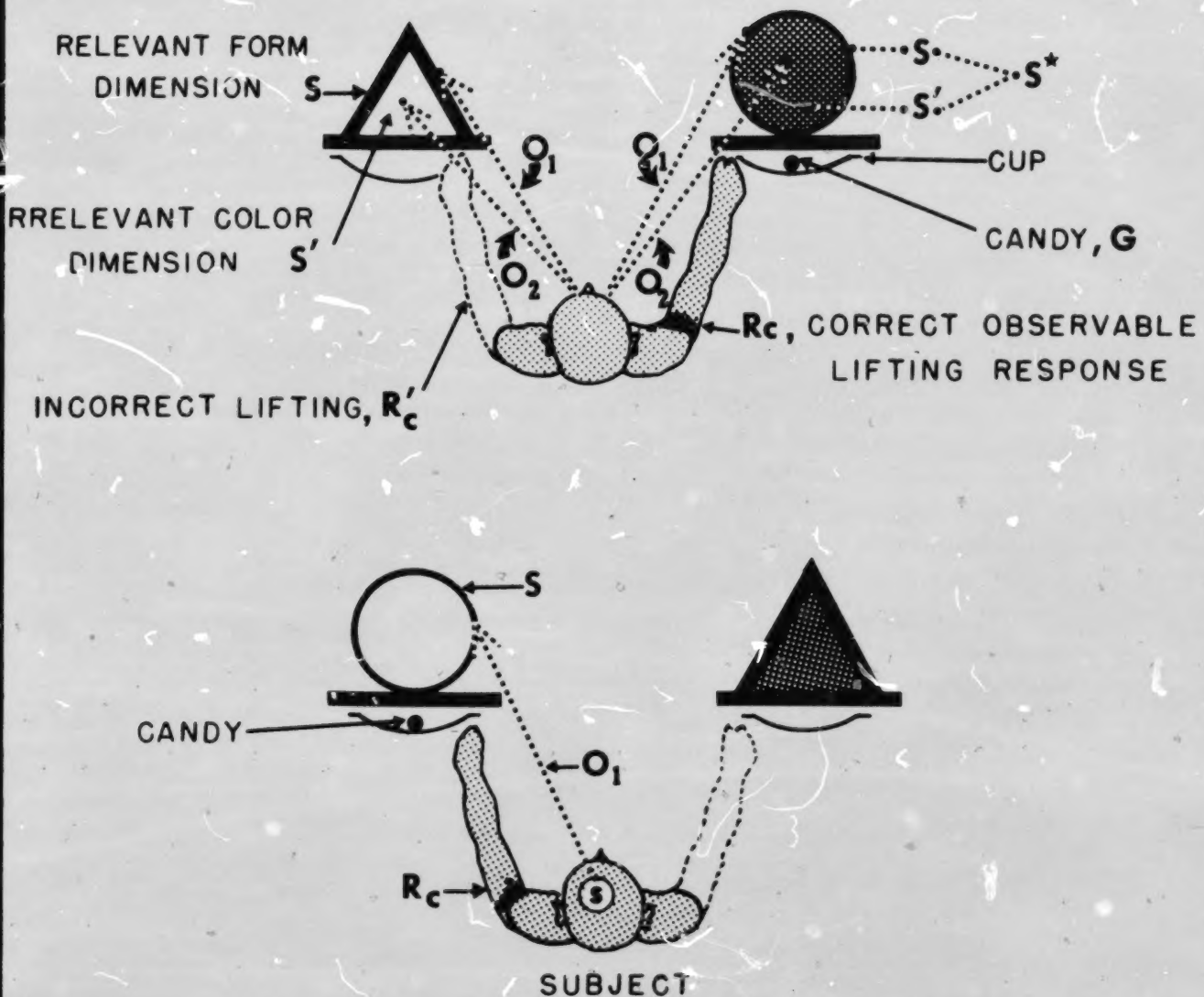


Figure 7. A diagram of some of the events of the theory. At top a yellow triangle and blue circle are presented simultaneously in a Form Discrimination Problem. At bottom the arrangement of stimuli on another trial is shown with candy following the relevant form cue. Approach to an irrelevant component gets reinforced randomly on half the trials.

The circle inside the subject's head represents the transformation by O_1 of the form dimension S to the effective cue s_1 (circle). Theoretically, the observation of the triangle could be substituted with no difference in outcome.

Other aspects of the diagram are described in the text.

The diagram does not make clear that while dimensional aspects of stimuli elicit attention, it is the result of attention, specific cues, that elicit the instrumental responses. But the distinction between dimensions and cues is a central one for the theory and deserves preliminary elaboration.

Dimensions are broad classes of cues having a common discriminative property. Dimensions have the lower informational content of a stimulus display, cues the higher. Consider color and form as the instances of dimensions given in the diagram. If the subject attends to color, he will see that yellow and blue are the specific cues present. If he observes form, a circle and triangle become effective cues. Changing the colors to red and green or the forms to square and cross does not disturb the observing responses because the dimensions of form and color are still intact. All colors have the common property of being colored, and it is this common property which the observing response transforms to specific cues. The precise physical description and mechanism of reception of the color dimensional property are in principle not insoluble problems in psychophysical and physiological psychology. As we will demonstrate later, dimensional properties are also discoverable in the context of discrimination learning experiments, especially by the experimental operation of intradimensional shifts. If, for example, training a subject to attend to color when blue and yellow are positive and negative cues leads to positive transfer when red and green are substituted in an intradimensional shift test, the inference of a color dimension is supported. As a corollary, knowledge of the existence of a dimension permits the demonstration of the negative transfer of extradimensional shift. As illustration, changing from form relevant to position relevant, or vice-versa, will lead to negative transfer to the extent that form and position are different dimensions.

It should be clear that stimulus dimensions are major constructs of the present theory. Things tentatively identified as dimensions can be manipulated physically to test a number of theoretical predictions, the accuracy of which serve as indicators of the truth of the theory in general and of the identification in particular. Thus, discovery of dimensions is part of the business of confirming the theory, and both are quite programmatic. Some instances of dimensions for our retardate population, we posit to be form, color, position, brightness, and size.

Some remarks about response specification will complete this brief qualitative description of the theory. For the simpler models of the theory, all responses are mutually exclusive (competing) and exhaustive (one must occur). The number of possible observing responses can vary from 1 to n, but only one is assumed to occur immediately before the moment of choice. This assumption identifies our "One-look" model. "Two-look" models can also be built when this assumption is relaxed, and the possibilities of generalization to an "N-look" model are considered. A simple case of an N-look model may be obtained merely by redefining O_1 . If some set of dimensions is always observed at the same time, as a unit, and the same values always presented together by the experimenter, the predictions of the model with reference to this multiple-look would be the same as for a simple "one-look" O_1 .

The two-choice experimental situation requires the inference of just two competing instrumental responses, only one of which occurs at the moment of choice. The theory can be generalized to apply to experiments with more choices without great difficulty, but we have not chosen to explore such models yet, since all our data are from two-choice experiments.

A final line of generalization we have considered is in the area of schedules of partial reinforcement. The methods for introducing the now traditional parameter (probability of reinforcement of some reference response) are the same as those used by Estes and Straughan (1954) and Bush and Mosteller (1955).

Quantitative Aspects of Theory

Mathematical description of the theory is aided by construction of the probability tree in Figure 8. Reading the tree from left to right, we see

Insert Fig. 8 about here

that each trial begins with presentation of S^* , which elicits just one of the n competing observing responses O_i with probabilities $Po(i)$. The observing response, whichever occurs, exposes the specific cues of the dimension observed. These in turn elicit one of two instrumental approach responses with conditional probabilities $Pr(i)$ and $1-Pr(i)$. Given the occurrence of the relevant observing O_i , reinforcement (G) always follows approach to the positive cue, with a standard 100% schedule. Given any of the irrelevant observing responses, the rules of experimentation ensure that the probability of reinforcement is $\frac{1}{2}$ regardless of what instrumental reaching response is made.

The tree shows $4n-2$ possible, alternative sequences that may comprise a trial. The next step in theory construction associates with each outcome a rule for changing the probabilities dependent upon the different outcomes.

Direct reinforcement. - If a trial ends in reinforcement, the probabilities of both responses occurring on that trial (observing and instrumental) undergo increments proportional to their complements. The constants of proportionality (θ) may differ for the instrumental and observing responses. The change in probability may be expressed in equation form: $\Delta \text{Prob.} = +\theta (1-\text{Prob.})$.

Direct extinction. - For a trial ending in nonreinforcement, probabilities of both responses in the chain occurring on that trial undergo decrements proportional to their magnitudes. The constants of proportionality (θ) are set equal to those of acquisition, as a first approximation. In equation form: $\Delta \text{Prob.} = -\theta (\text{Prob.})$.

Indirect reinforcement. - When a trial ends in extinction, the probabilities of all the nonelicited observing responses gain an increment, each one getting a fraction of the probability lost by the directly extinguished observing response. These fractions are such as to preserve the relative strengths of the nonelicited O_i . The fractions can be expressed in symbolic terms as $Po(i)/(1-Po(j-))$, if we let $Po(j-)$ be the probability of the directly extinguished observing response, and $Po(i)$ be the probability of any one of the remaining, nonelicited observing responses.

For the conditional probabilities of the instrumental responses, any loss in the probability of R_i by extinction is accompanied by an equal gain in the probability of R_i' , since the two probabilities are complements.

Indirect extinction. - The analogous rule holds here. Direct gains in attention to one dimension are spontaneously accompanied by changes in the strengths of other observing responses - each in proportion to its relative strength. Rules of this kind are necessary to conserve probability in a competing response system.

Summary of rules for probability change. - The rules described above can be redescribed in operator form to facilitate tabular summary of their application. The acquisition operator Q_A is defined by the equation,

$$Q_A (\text{Prob.}) = (\text{Prob.}) + \theta (1-\text{Prob.});$$

and the extinction operator Q_E by

$$Q_E (\text{Prob.}) = (\text{Prob.}) - \theta (\text{Prob.}).$$

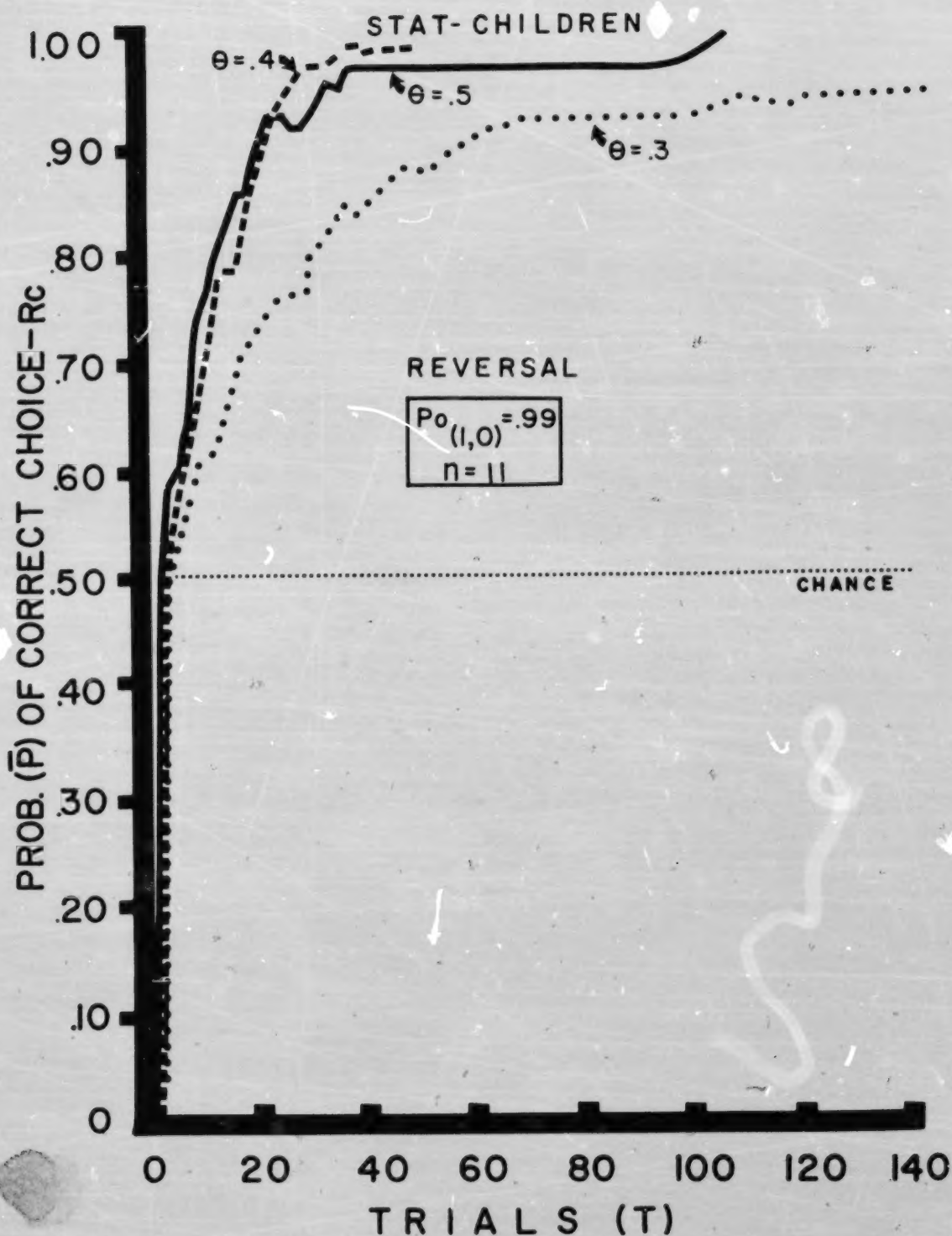


Figure 8. Probability tree of the basic model. The branches represent all the possible events that can theoretically occur on a single trial. The probability of occurrence of each branch is drawn in brackets.

The set of relevant and irrelevant stimulus dimensions is denoted S^* . Other notation is defined in the text.

The indirect acquisition operator Q_{aj} is defined by

$$Q_{aj}(Po_{(1)}) = Po_{(1)} + \frac{\theta_o \cdot Po_{(1)} \cdot Po_{(j-)}}{1 - Po_{(j-)}},$$

and the indirect extinction operator Q_e by

$$Q_e(Po_{(1)}) = Po_{(1)} - \frac{\theta_o Po_{(1)} \cdot (1 - Po_{(j+)})}{1 - Po_{(j+)}} = Po_{(1)} - \theta_o \cdot Po_{(1)},$$

where $Po_{(j+)}$ is the probability of the directly reinforced observing response. Note that Q_e turns out to be identical to Q_g . The occasions for application of the operators together with the probabilities of application are summarized in Table 1.

Insert Table 1 about here

Entries have not been tabulated for sequences beginning with observing responses between Q_2 and Q_n in the probability tree, since these would not differ from those of Q_2 and Q_n . Q_I denotes the identity operator. The probability of each sequence has been included to help show the derivation of Stat-children and expected operators.

Probability of a correct overt response.— From the probability tree of Figure 8 can be derived an equation for the probability (P) of R_c , the correct overt response of choosing the stimulus object associated with the positive cue. Summing up the probability products of all branches leading to G yields the basic equation:

$$P = Po_{(1)} \cdot Pr_{(1)} + \frac{1}{2}(1 - Po_{(1)}) \quad \dots \text{Equation 1}$$

P is the dependent variable of the system, and Equation 1 relates it to the probability of observing the relevant dimension and the conditional probability of approaching the positive cue given observation of the relevant dimension.

The Problem of Approximations

The theory so far consists of (a) Equation 1 which relates the observable dependent variable P to probabilities of two unobservable responses, and (b) rules for changing probabilities of these inferred responses on each trial, for a single subject. This is not enough for empirical tests of the theory. Our data are usually in the form of learning curves relating number of training trials (T) to percentage of correct responses (R_c) of a group of subjects, but the theory says nothing about these variables yet.

If for a large group of identical subjects, we know all the starting probabilities in the tree (Figure 8) and the other parameter values (θ_s and θ_o) we can compute directly the mean probability P_1 of the group on trial 1, and, using standard methods, the expected value P_2 for trial 2.

$$\begin{aligned} \bar{P}_2 = & \left[Po_{(1)} \cdot Pr_{(1)} \right] \left[Q_A Po_{(1)} \cdot Q_A Pr_{(1)} + \frac{1}{2}(1 - Q_A Po_{(1)}) \right] \\ & + \left[Po_{(1)} \cdot (1 - Pr_{(1)}) \right] \left[Q_E Po_{(1)} \cdot Q_A Pr_{(1)} + \frac{1}{2}(1 - Q_E Po_{(1)}) \right] \\ & + \left[1 - Po_{(1)} \right] \left[Pr_{(1)} \cdot \frac{1}{2}(Q_e Po_{(1)} + Q_s Po_{(1)}) + \frac{1}{2}(1 - \frac{1}{2}(Q_e Po_{(1)} \right. \\ & \left. - Q_s Po_{(1)})) \right] \end{aligned}$$

Table 1
Operators Applied to Four Probabilities
Given the Various Sequences of Events
That Can Occur on a Trial

Sequences of Events	Probability of Sequence	Operators Applied To			
		Po ₍₁₎	Po ₍₂₎	Po _(n)	Pr ₍₁₎
O ₁ R ₁ G	Po ₍₁₎ Pr ₍₁₎	Q _A	Q _e	Q _e	Q _A
O ₁ R ₁ 'G'	Po ₍₁₎ (1-Pr ₍₁₎)	Q _E	Q _{a1}	Q _{a1}	Q _A
O ₂ R ₂ G	Po ₍₂₎ Pr ₍₂₎ ^{1/2}	Q _e	Q _A	Q _e	Q _I
O ₂ R ₂ G'	Po ₍₂₎ Pr ₍₂₎ ^{1/2}	Q _{a2}	Q _E	Q _{a2}	Q _I
O ₂ R ₂ 'G	Po ₍₂₎ (1-Pr ₍₂₎) ^{1/2}	Q _e	Q _A	Q _e	Q _I
O ₂ R ₂ 'G'	Po ₍₂₎ (1-Pr ₍₂₎) ^{1/2}	Q _{a2}	Q _E	Q _{a2}	Q _I
O _n R _n G	Po _(n) Pr _(n) ^{1/2}	Q _e	Q _e	Q _A	Q _I
O _n R _n G'	Po _(n) Pr _(n) ^{1/2}	Q _{an}	Q _{an}	Q _E	Q _I
O _n R _n 'G	Po _(n) (1-Pr _(n)) ^{1/2}	Q _e	Q _e	Q _A	Q _I
O _n R _n 'G'	Po _(n) (1-Pr _(n)) ^{1/2}	Q _{an}	Q _{an}	Q _E	Q _I

The result is a theoretical learning curve for two trials for a group of identical subjects with equal irrelevant observing response probabilities. When one tries to compute $\bar{P}_3, \bar{P}_4, \dots, \bar{P}_{100}$, however, the work rapidly becomes prohibitive. While it may not be impossible in principle to find an explicit solution for the desired function $\bar{P}(T) = f(T)$ for this system, the methods are not known to us. For this reason, we make use of statistical approximation procedures - Stat-children and, also, a kind of expected operator approximation.

Stat-children.- Bush and Mosteller (1955) have described in detail a "Monte Carlo" method for getting approximate solutions of stochastic models. We employ these methods to run Stat-children. A Stat-child is a statistical creature that obeys the rules of a probability model. If a response is to be elicited with a given probability, the Stat-child consults a table of random numbers in its head to translate that probability into a yes-no decision. For our model, computation of the behavior of Stat-children requires (a) the operators listed in Table 1, (b) a table of random numbers to convert the probabilities of Column 2 in Table 1 to yes-no decisions on application of the operators, (c) a set of starting probabilities, θ values and a value of n , and (d) Equation 1.

Populations of Stat-children are conveniently run on highspeed digital computers because of the tedious nature of the computations required. Major results include the proportions of Stat-children making the correct overt response on each trial for populations having various starting probabilities and parameter values. These proportions serve as estimates of $\bar{P}(T)$.

Expected operator approximation of $\bar{P}_{o(1,T)}$. - If each of the operators listed under $\bar{P}_{o(1)}$ in Table 1 is weighted by the probability of its application (second column) and applied to $\bar{P}_{o(1,T)}$, the sum of these weighted operations yields an expected operator approximation of $\bar{P}_{o(1,T+1)}$. Using the relation

$$\Delta \bar{P}_{o(1,T)} = \bar{Q}\bar{P}_{o(1,T)} - \bar{P}_{o(1,T)}$$

we then write an expression for the expected change in the mean probability of a relevant observing response.

$$\begin{aligned} \Delta \bar{P}_{o(1)} = & \bar{P}_{o(1)} \cdot \bar{P}_{r(1)} \theta_o(1 - \bar{P}_{o(1)}) + \bar{P}_{o(1)} \cdot (1 - \bar{P}_{r(1)}) \cdot (-\theta_o \bar{P}_{o(1)}) \\ & + \bar{P}_{o(2)} \cdot \frac{1}{2}(-\theta_o \bar{P}_{o(1)}) + \bar{P}_{o(2)} \cdot \frac{1}{2}(\theta_o \bar{P}_{o(2)}) \cdot \frac{\bar{P}_{o(1)}}{1 - \bar{P}_{o(2)}} \\ & + \dots + \bar{P}_{o(n)} \cdot \frac{1}{2}(-\theta_o \bar{P}_{o(1)}) + \bar{P}_{o(n)} \cdot \frac{1}{2}(\theta_o \bar{P}_{o(n)}) \cdot \frac{\bar{P}_{o(1)}}{1 - \bar{P}_{o(n)}} \end{aligned}$$

which simplifies to

$$\Delta \bar{P}_{o(1)} = (\theta_o/2) \bar{P}_{o(1)} \cdot (2\bar{P}_{r(1)} - 1 - n + \sum_{i=2}^n \frac{1}{1 - \bar{P}_{o(i)}})$$

... Equation 2.

Applying the expected operator $\bar{Q}$ to a mean probability of a group of subjects with different probability values is not the usual way of obtaining an expected mean and the result may be a quite biased estimate of $\bar{P}_{o(1,T)}$. Calculation of the expected operator for our system in the usual way leads to fearsome complexities of the kind Bush and Mosteller (1955) have shown for simpler models with subject-controlled events. The accuracy of the approximation achieved by our present method can be assessed to some extent by comparing results with mean Stat-child performance.

Approximation of the expected change in mean probability of irrelevant observing responses. - The same logic used to compute $\Delta \bar{P}_o(1)$ can be used to approximate $\Delta \bar{P}_o(i)$ ($i = 2, 3, \dots, n$). Let $\bar{P}_o(1)$ be any particular irrelevant $\bar{P}_o(1)$ chosen as reference. The expected change in the mean probability of observing the j th irrelevant dimension is

$$\Delta \bar{P}_o(j) = (\theta_o/2) \cdot \bar{P}_o(j) \left[\frac{2\bar{P}_o(1) (\bar{P}_o(-1) - \bar{P}_r(1))}{1 - \bar{P}_o(1)} + 2\bar{P}_o(1) + 1 - n + \sum_{\substack{i=2 \\ i \neq j}}^n \frac{1}{1 - \bar{P}_o(i)} \right] \quad \dots \text{Equation 3}$$

Approximation of the expected change in relevant instrumental response probability. - We reason as before to get

$$\Delta \bar{P}_r(1) = \bar{P}_o(1) \cdot \bar{P}_r(1) \cdot \theta_r(1 - \bar{P}_r(1)) + \bar{P}_o(1) \cdot (1 - \bar{P}_r(1)) \theta_r(1 - \bar{P}_r(1)) - \bar{P}_o(1) \cdot \theta_r(1 - \bar{P}_r(1)) \quad \dots \text{Equation 4}$$

Expected change in the mean probability of a correct response. - If equation 1 holds for the individual subject, then

$$\bar{P} = \bar{P}_o(1) \cdot \bar{P}_r(1) + \frac{1}{2}(1 - \bar{P}_o(1)) \quad \dots \text{Equation 5}$$

does not hold exactly for a group, but it does yield an approximation. This, in effect, adds an approximation on top of an approximation, but it permits us to write the corresponding difference equation

$$\Delta \bar{P} = \bar{P}_o(1) \cdot \bar{P}_r(1) + \bar{P}_r(1) \cdot \bar{P}_o(1) + \bar{P}_o(1) \cdot \bar{P}_r(1) - \Delta \bar{P}_o(1) \quad \dots \text{Equation 6}$$

Into Equation 6 are then substituted values of $\Delta \bar{P}_o(1)$ and $\Delta \bar{P}_r(1)$ computed by means of Equation 2, 3, and 4.

Starting from assumed initial conditions $\bar{P}_o(1)$ and $\bar{P}_r(1)$, Equation 6 can be stepped along trial by trial to given an approximation of $\bar{P}(T)$ for various values of parameters θ_o , θ_r , and n .

Explicit solutions for any of the nonlinear difference equation presented here are not known so we have used a digital computer again for trial-by-trial calculations. These calculations use less machine time than the corresponding Stat-child computations.

There is little danger of misinterpreting these approximations to an expected mean curve if it is recognized that $\bar{P}(T+1)$ equals exactly the expected mean of a group of subjects if all have the same values - $\bar{P}_o(i, T)$ and $\bar{P}_r(i, T)$ - on the preceding trial, T , and if all receive the average Δs on trial T . Such approximations can provide useful information about the properties of the system, even though they may be unable to describe the behavior of groups of subjects varying widely in $\bar{P}_o(i, T)$ and $\bar{P}_r(i, T)$.

Theoretical Effects of Parameters

Initial $\bar{P}_o(1)$ and n . - The probability of attending to the relevant stimulus dimension at the very beginning of training ($\bar{P}_o(1, 0)$) may vary, and Figure 9 shows the theoretical effects on group discrimination learning curves. Each function represents the mean probability of correct choice (R_c) for 20

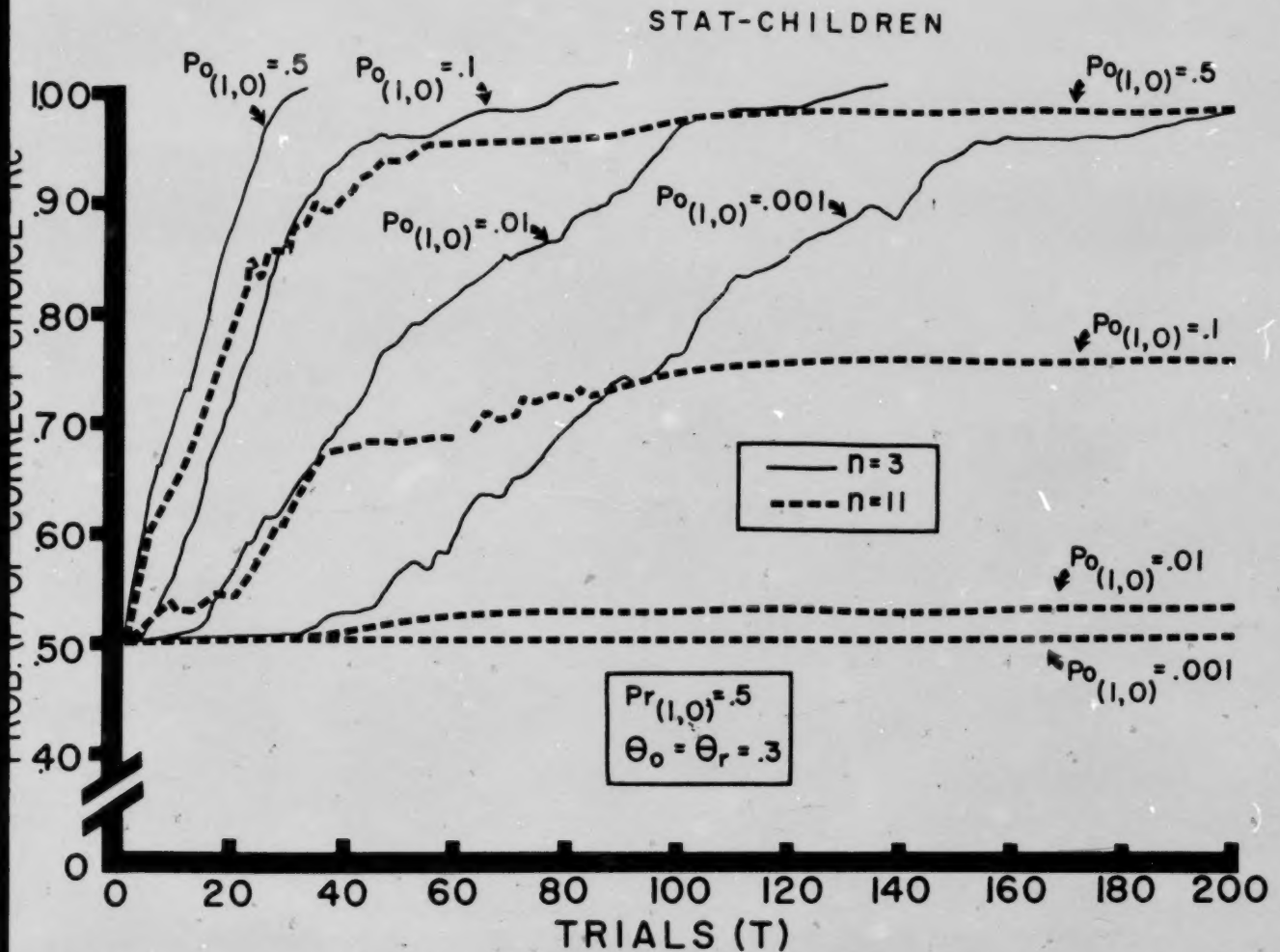


Figure 9. Mean discrimination curves for 8 groups of 20 Stat-children. Two families of functions are generated by the n parameter: the solid line is for $n=3$, dotted for $n=11$. Within each family a separate curve is plotted for different values of $P_0(1,0)$. For all functions, the fixed parameters are $Pr(1,0) = .5$, $\theta_0 = \theta_r = .3$, and all starting probabilities for irrelevant observing responses are equal within a condition.

Stat-children. The total number of dimensions (n) appears as a parameter. For $n=3$ (one relevant dimension and two irrelevant) the effect of initial probability of a relevant observing response is to change the shape of the mean performance curves from a fast rising, negatively accelerated function (when $Po_{(1,0)} = .5$), to a slower rising, ogival function (when $Po_{(1,0)} = .001$). For $n = 11$, the forms of the functions remain roughly the same, but the rates are markedly attenuated. All functions are asymptotic at unity, given enough trials.

Comparisons of all curves in Figure 9 show clearly that the two parameters $Po_{(1,0)}$ and n interact strongly. The larger the value of n , the greater the tendency of a low initial $Po_{(1)}$ to retard the rate of improvement of performance with training.

Effects of θ . - (a) Equal θ values. The effect of variation in θ (when $\theta_o = \theta_r = \theta$) can be seen in the differences among Figures 9, 10, and 11. These result from the change in θ from .3 (Figure 9) to .5 (Figure 10) to .7

Insert Fig. 10 about here

(Figure 11).

Insert Fig. 11 about here

All three parameters $Po_{(1,0)}$, n , and θ interact. One way to describe the interaction is as follows: the effect of $Po_{(1,0)}$ on performance is controlled by n , and this control is, in turn magnified by low θ values.

(b) Unequal θ values. Equal θ s are merely a simplification and not required by theory. Figure 12 holds the instrumental rate parameter θ_r constant at .5 and allows θ_o to vary from .05 to .95. Other parameter values are shown

Insert Fig. 12 about here

in the figure. Reversing the procedure, Figure 13 holds the observing response parameter θ_o constant and allows θ_r to vary. Note that the curves for

Insert Fig. 13 about here

$\theta_o = .95$ and .7 are not in the expected order. This is random error and indicates that Stat-children, like ordinary children, are sometimes misbehaved. Correcting such errors requires the mean of a larger number of subjects.

Unequal irrelevant $Po_{(1,0)}$. - For previous theoretical curves, values of irrelevant observing response probabilities have all been equal within a condition. If, for example, there were three dimensions, with the relevant dimension having a starting probability of .1 of being observed, then the two irrelevant observing response probabilities - $Po_{(2,0)}$ and $Po_{(3,0)}$ - would be equal at .45. This condition is plotted in Figure 14 with other parameters as indicated. If the ratio of $Po_{(2,0)}$ to $Po_{(3,0)}$

Insert Fig. 14 about here

STAT-CHILDREN

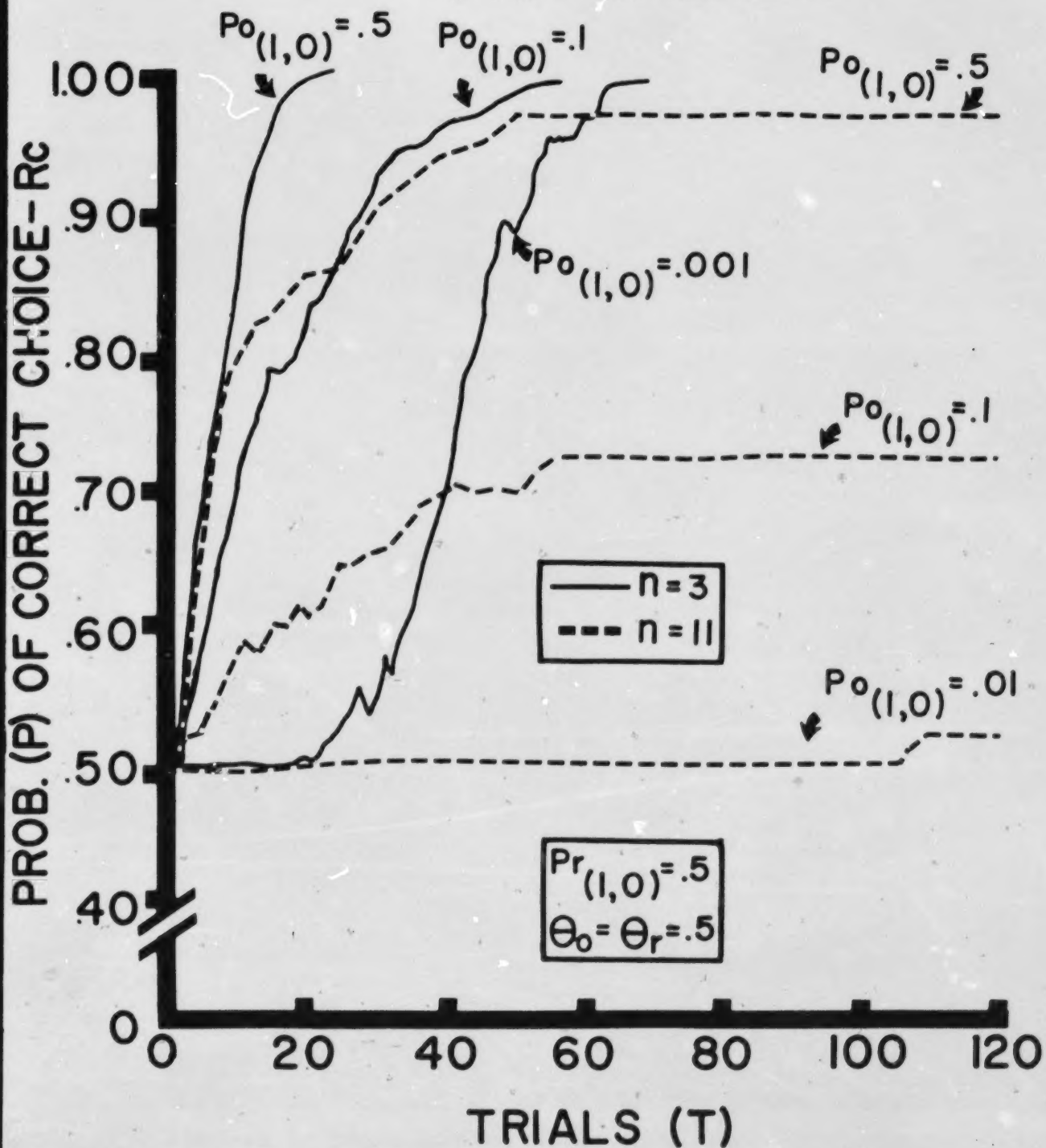
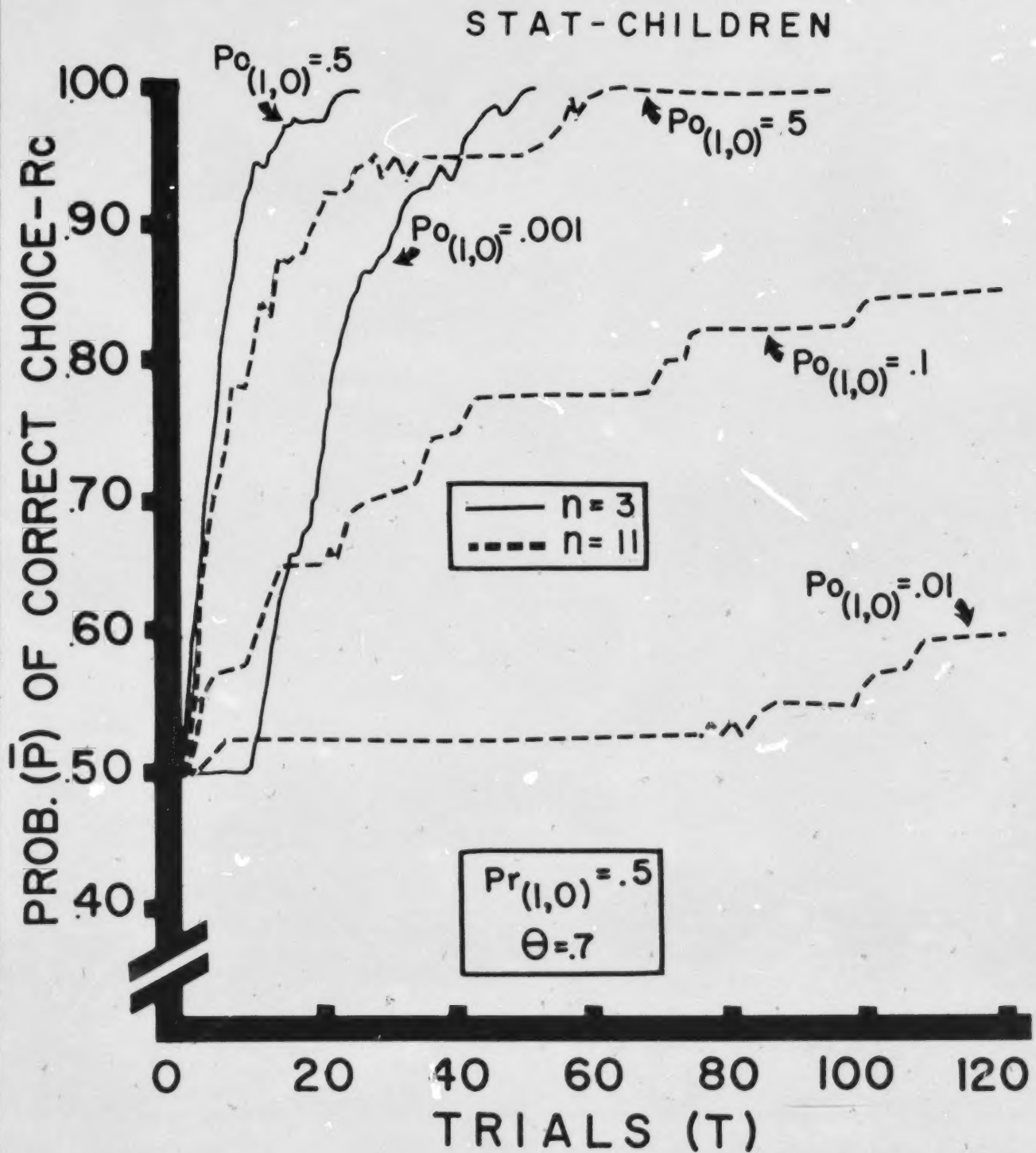


Figure 10. Theoretical discrimination learning curves corresponding to those of Figure 9, but with $\theta = .5$.

Figure 11. θ value equal to .7.

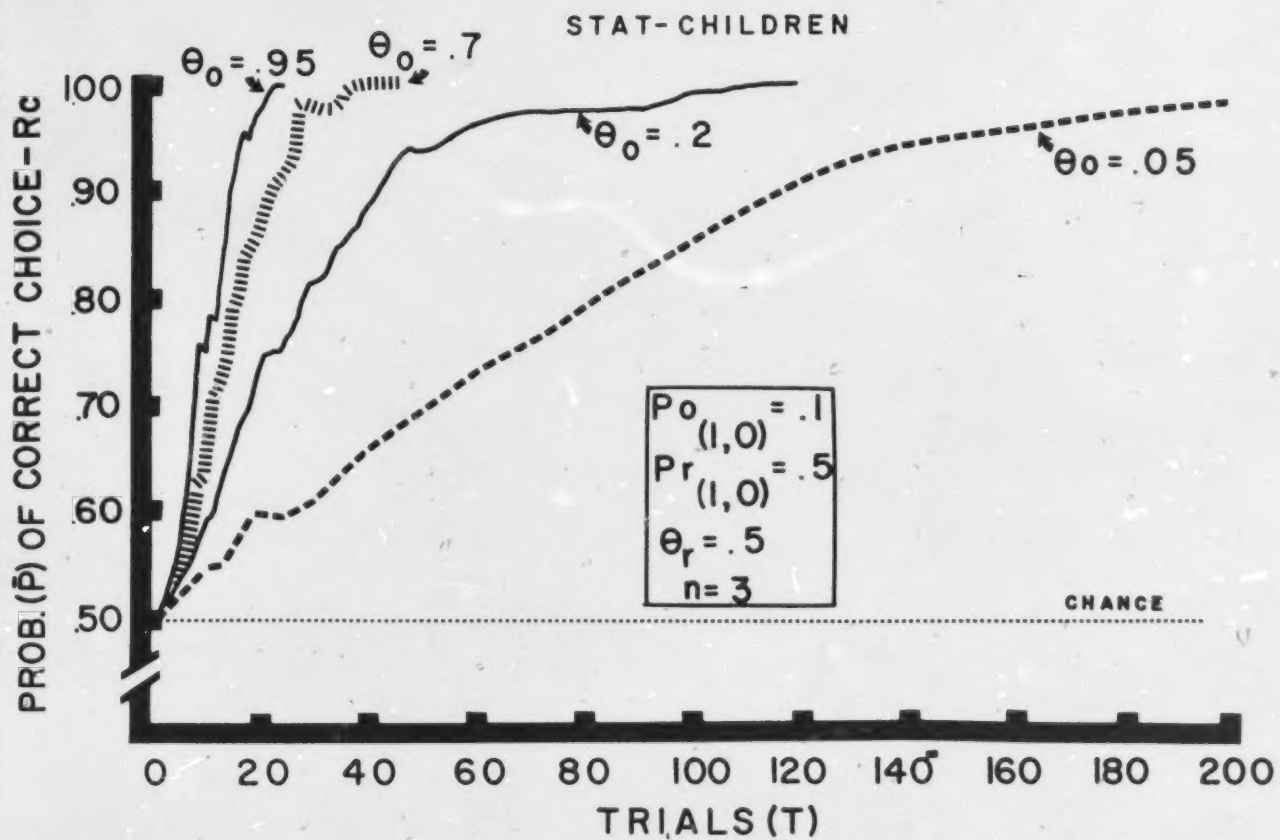


Figure 12. Effects of variation in the observing response parameter θ_0 .

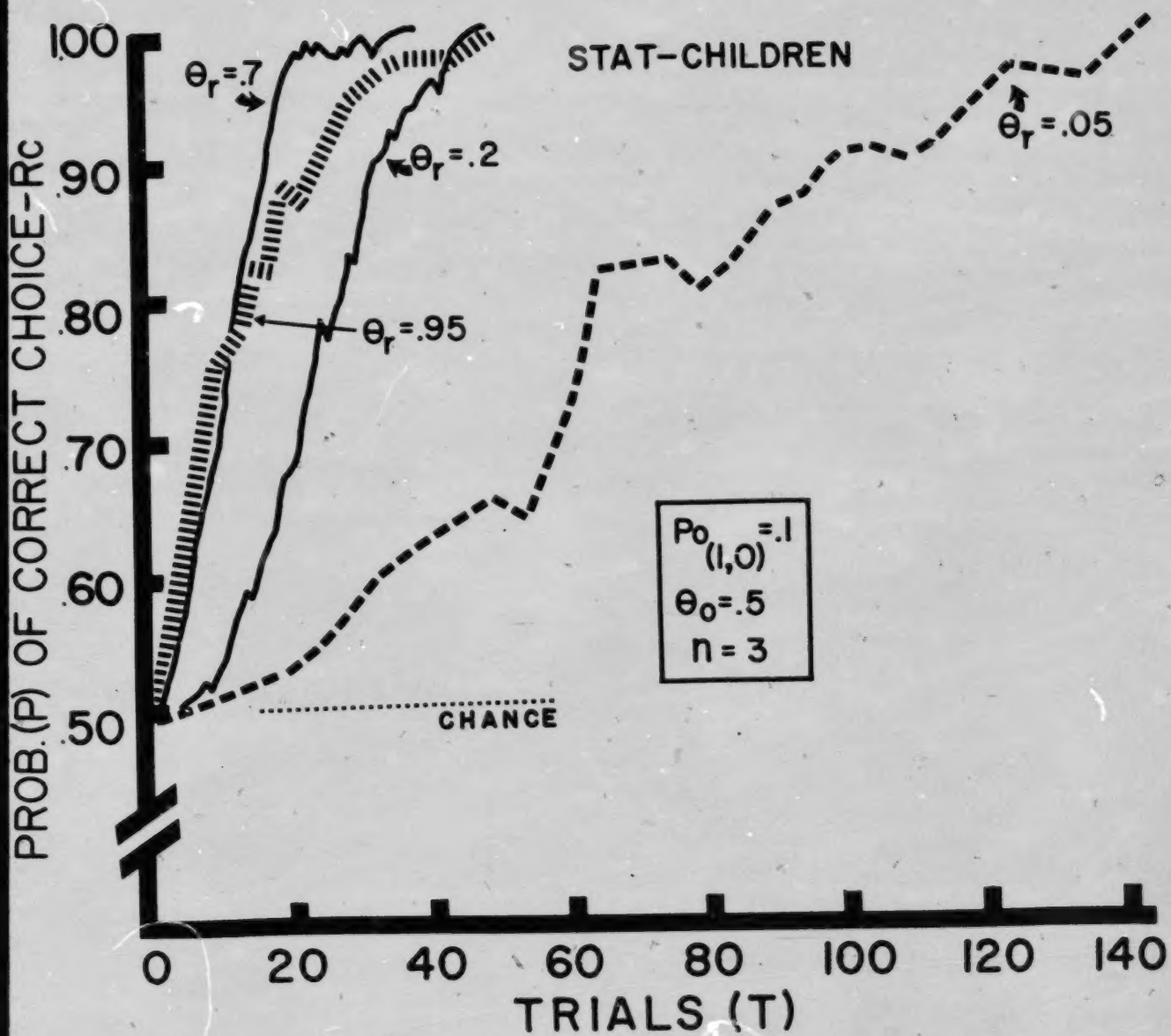


Figure 13. Changing the parameter θ_r .

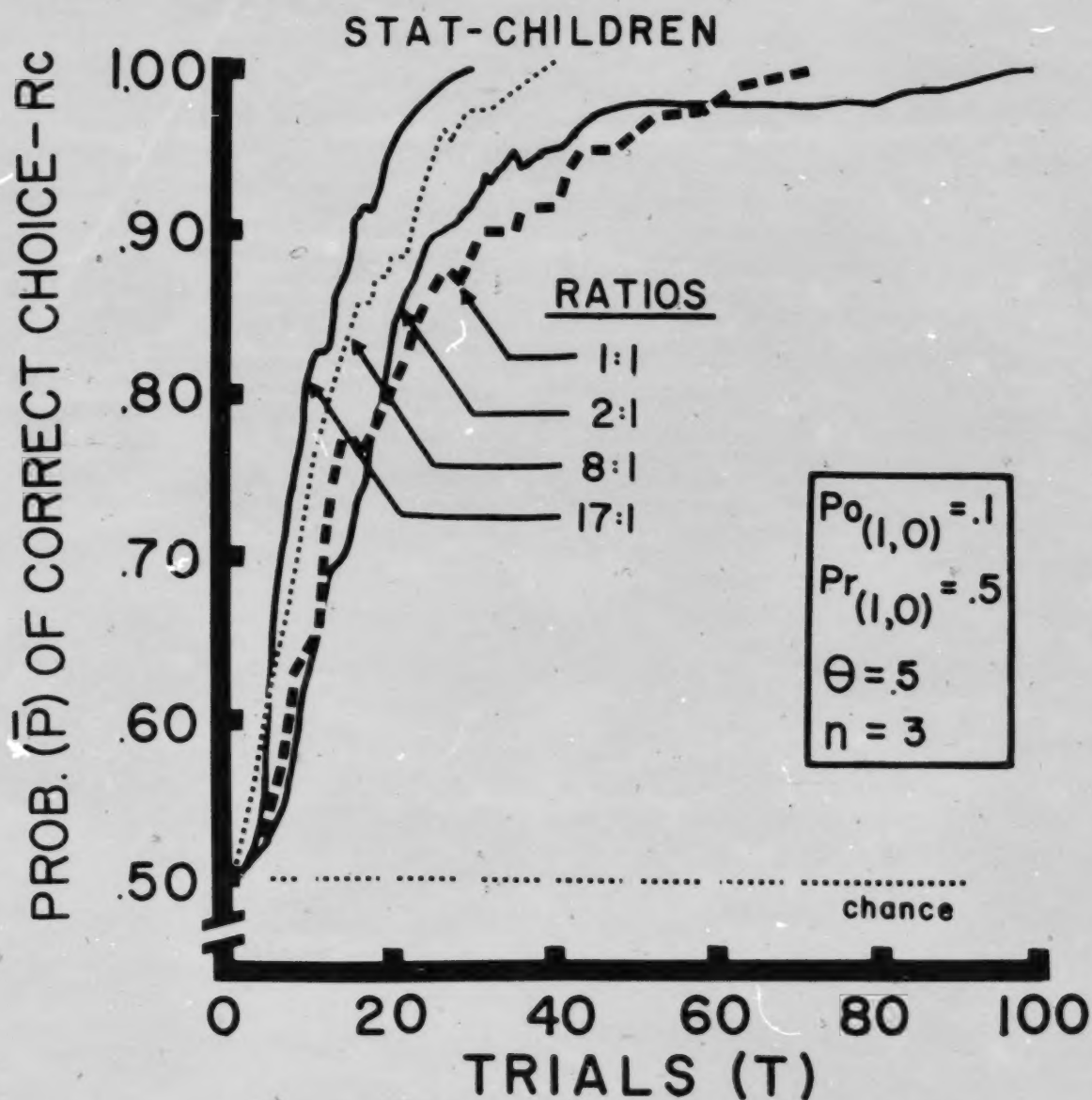


Figure 14. Effects of Stat-children performance of unequal ratios of irrelevant observing response probabilities, with other parameters constant.

$P_o(2,0)$ and $P_o(3,0)$ equal .6 and .3 for the 2:1 ratio, equal .8 and .1 for the 8:1 ratio, and equal .85 and .05, respectively, for the 17:1 ratio.

is changed to 8:1, holding everything else constant, Figure 14 shows the rate of learning to be enhanced. This result may seem surprising. One might expect that the competition of a relatively strong irrelevant observing response would interfere with the growth of the relevant observing response. But the theory predicts the opposite. Increasing the ratio to 17:1 makes the learning even faster. This interesting prediction of the model is testable with carefully arranged transfer paradigms.

Pr_(1,0) and reversal. - Under experimental conditions of original learning, the initial instrumental probability of approaching the positive cue, $Pr_{(1,0)}$, is at the chance value of .5, since the experimenter usually designates the positive cue randomly. There exists, however, a common transfer operation - simple discrimination reversal - that brings $Pr_{(1,0)}$ down to zero or approximately so, depending upon the strength of pre-reversal learning. Too large a number of families of reversal learning curves can be plotted for the various combinations of parameter values, so one set has been selected for its unusual properties and eventual relation to data. Figure 15 presents reversal learning curves for $n = 11$ and three values of θ . Initial probability of observing the relevant dimension is always high for reversals following learning to a stringent criterion, so $Po_{(1,0)}$ has been set at .99 in Figure 15.

Insert Fig. 15 about here

All the functions in Figure 15 rise quickly from the initial probability near zero at the start of reversal, reaching chance level in a trial or two after which the curves decelerate sharply. Although it may be difficult to see in Figure 15, these functions tend to flatten somewhat in the middle. More precisely, they begin with negative acceleration, change to positive, then revert again to negative. This tendency we have tentatively labelled a "reversal midplateau", perhaps a questionable label because the plateau is not always flat. Figure 16 is a re-plot of the first 10 trials of the $\theta = .5$ function with an expanded abscissa that accentuates the curvatures in question.

Insert Fig. 16 about here

The cause of the reversal midplateau is the extinction of the relevant observing response. The development of this inattention is shown graphically in Figure 17, a plot of the expected operator approximations of $P(T)$ along with $Po_{(1,T)}$ and $Pr_{(1,T)}$ for the parameters indicated. Note the rapid

Insert Fig. 17 about here

extinction of the relevant observing response during the early trials as the subject picks up the once-positive-but-now-negative stimulus.

The midplateau is strongly controlled by θ and n , and to some extent by starting probabilities. High values of θ together with low values of θ_r amplify the effect, as do large values of n . Figure 18 shows an extreme case.

Insert Fig. 18 about here

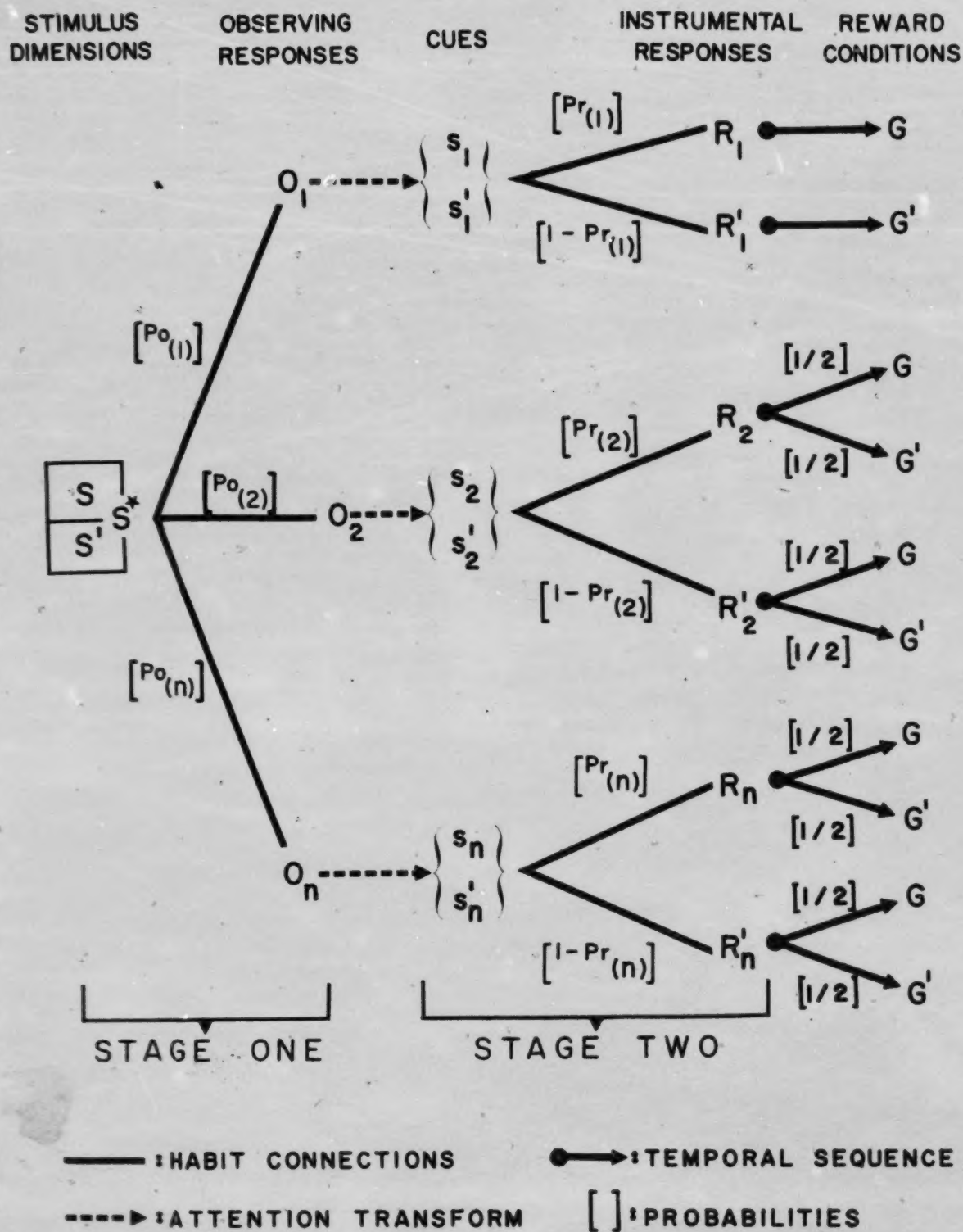


Figure 15. Reversal of a discrimination for three groups of 20 Stat-children with various θ values. $Pr(1,0)$ for all groups has been set at zero.

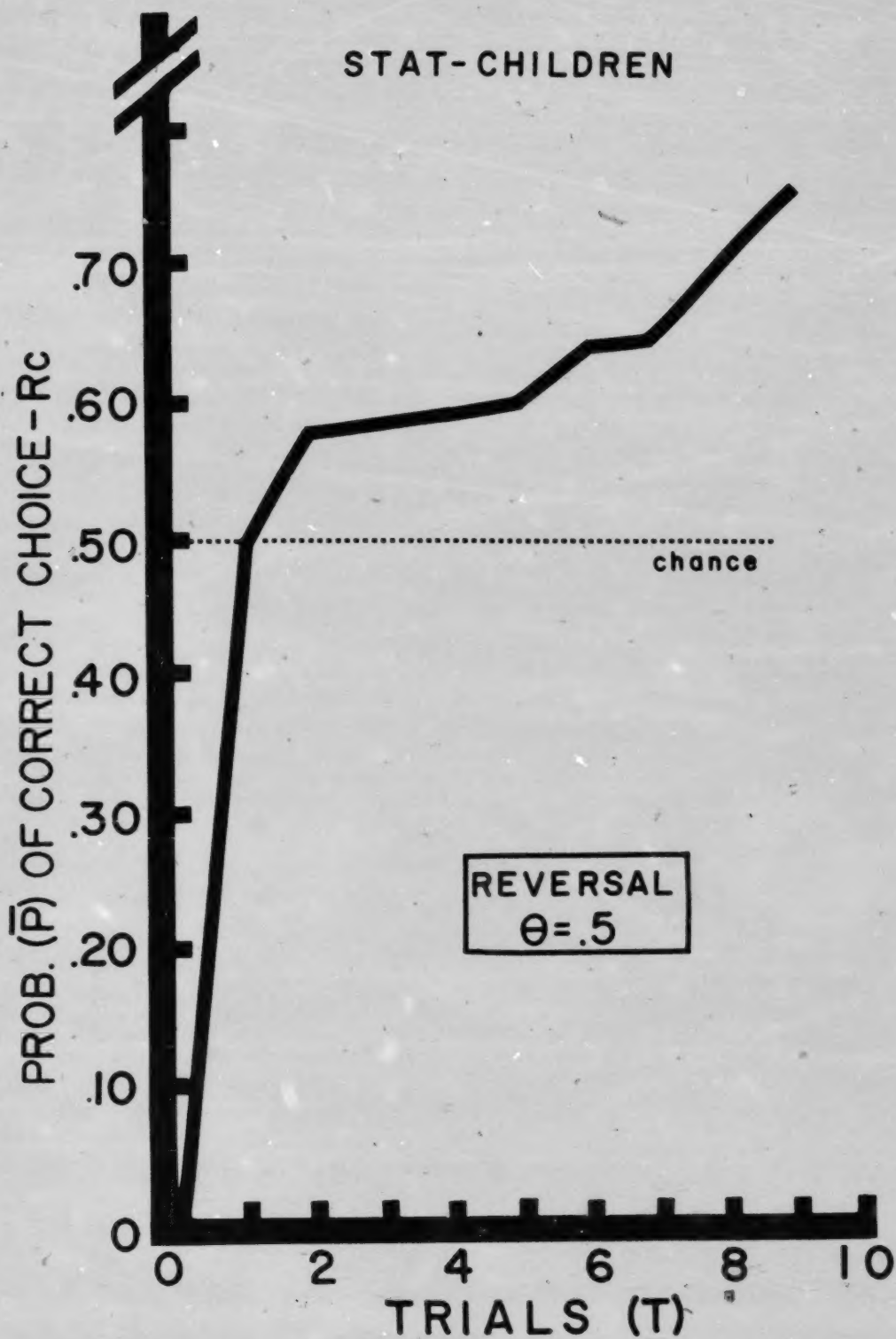


Figure 16. First 10 trials of the $\theta = .5$ function of Figure 15 replotted with an expanded abscissa.

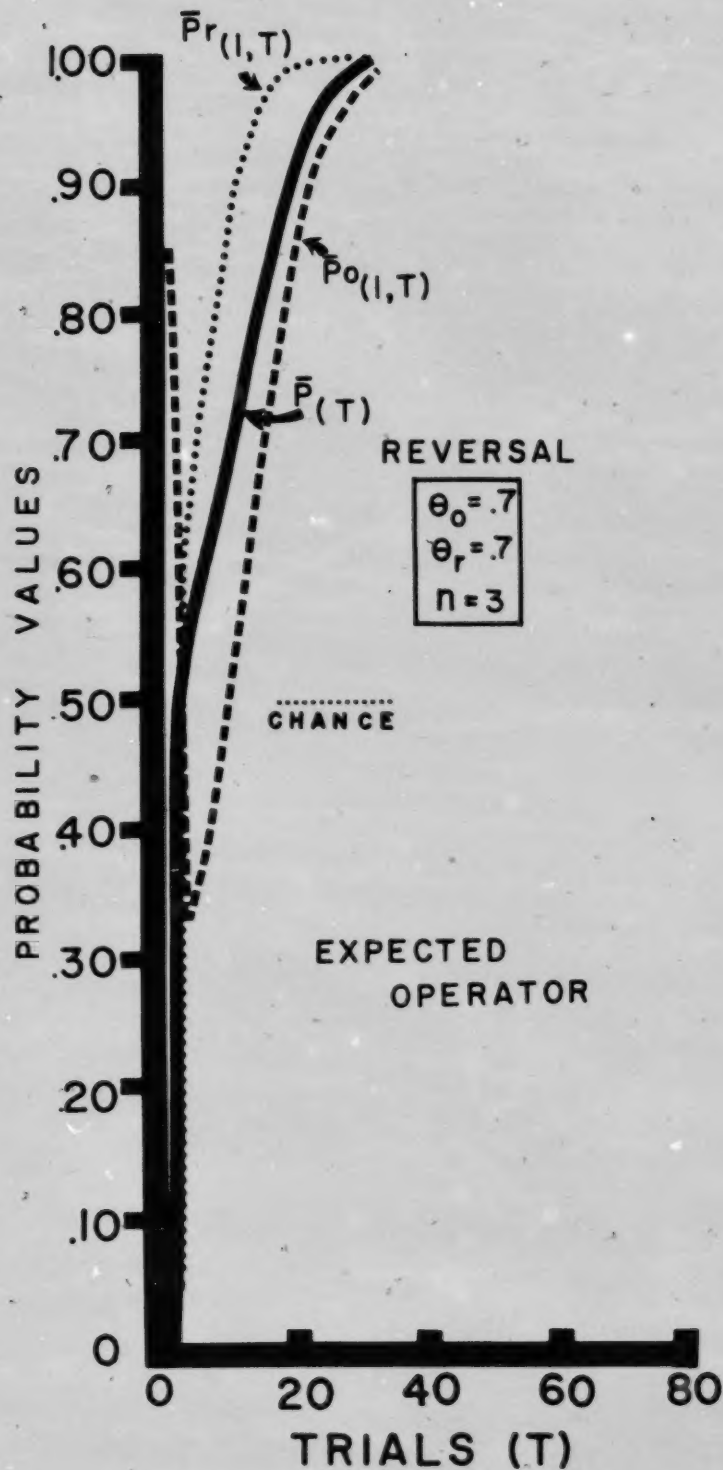


Figure 17. Expected operator approximation of theoretical reversal learning ($\bar{P}_T$). Also plotted are the theoretical determinants of ($\bar{P}_T$), the probabilities of a relevant observing response, $\bar{P}_o(1, T)$ and $\bar{P}_r(1, T)$, respectively. Starting values are those of Figure 15.

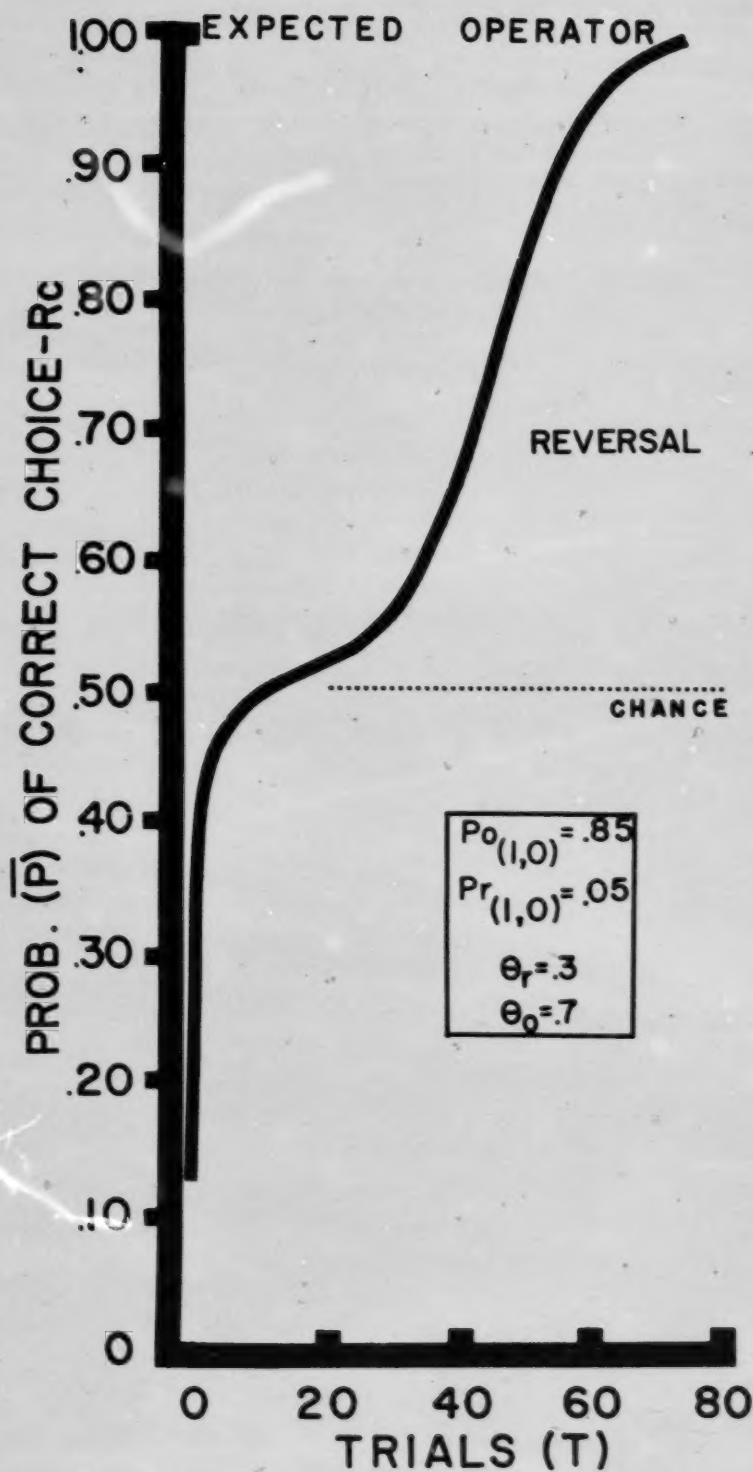


Figure 18. Expected operator approximation of a theoretical reversal function with parameter values that accentuate the reversal "midplateau."

The presence or absence of this unusual effect should be of assistance in the problem of parameter estimation, since only certain combinations of parameters predict it. Furthermore, we do not know of any other quantitative theories which predict this phenomenon, so its occurrence in data should have special probative value for the theory.

Variability

For each trial the computer program prints out a mean and standard deviation of the individual probabilities of a correct response for a group of Stat-children, and a series of zeros and ones indicating a correct or incorrect response for each Stat-child. Figure 19 shows the relation of these three measures.

Insert Fig. 19 about here

The function labelled "Zero-one" represents the proportion of Stat-children making the correct response. It is, of course, much more irregular than the mean probability curve, having an additional source of random variation. It corresponds, in this respect, more closely to raw data. Such correspondence, while desirable, makes comparison of trends difficult. Trial-by-trial variability of percentage correct response for retardate performance is often great enough to require averaging percentages over blocks of trials to reduce random variability and uncover trends. When this is done, we appropriately compare the data with the mean probability curve of Stat-children.

The random variability of mean Stat-children functions can be reduced by taking larger and larger samples. Figure 20 allows an estimate of the order of magnitude of reduction in random error by comparing the mean probability

Insert Fig. 20 about here

functions of four groups of 30, 50, 70, and 100 Stat-children. The maximum difference between the 30 and 100 cases is less than .10 for these arbitrarily selected parameter values. Since we are not yet at the stage of precise parameter estimation, we have not run large numbers of Stat-children under each of the various parameter conditions.

Backward Curves for Stat-Children

Theoretical effects of parameters, up to this point, have been shown using standard, forward learning curves. It is also possible to program a computer for backward Stat-children functions as well. We begin by showing the influence of differences in starting probabilities of observing the relevant stimulus dimension. For three widely different values of P_0 (1, 0), Figure 21 graphs backward learning curves for three groups of Stat-children reaching criterion. The position of the functions on the abscissa is that of the median subject. Note that the three functions have almost identical forms and, with the exception of lateral displacement, are almost equivalent. The contrast with forward learning curves (Figure 9) is marked.

The effects on backward curves of two other parameters, θ and n , appear in Figure 22. Four parameter conditions are presented for $n=3$ and $n=11$ and

Insert Figs. 21 and 22 about here

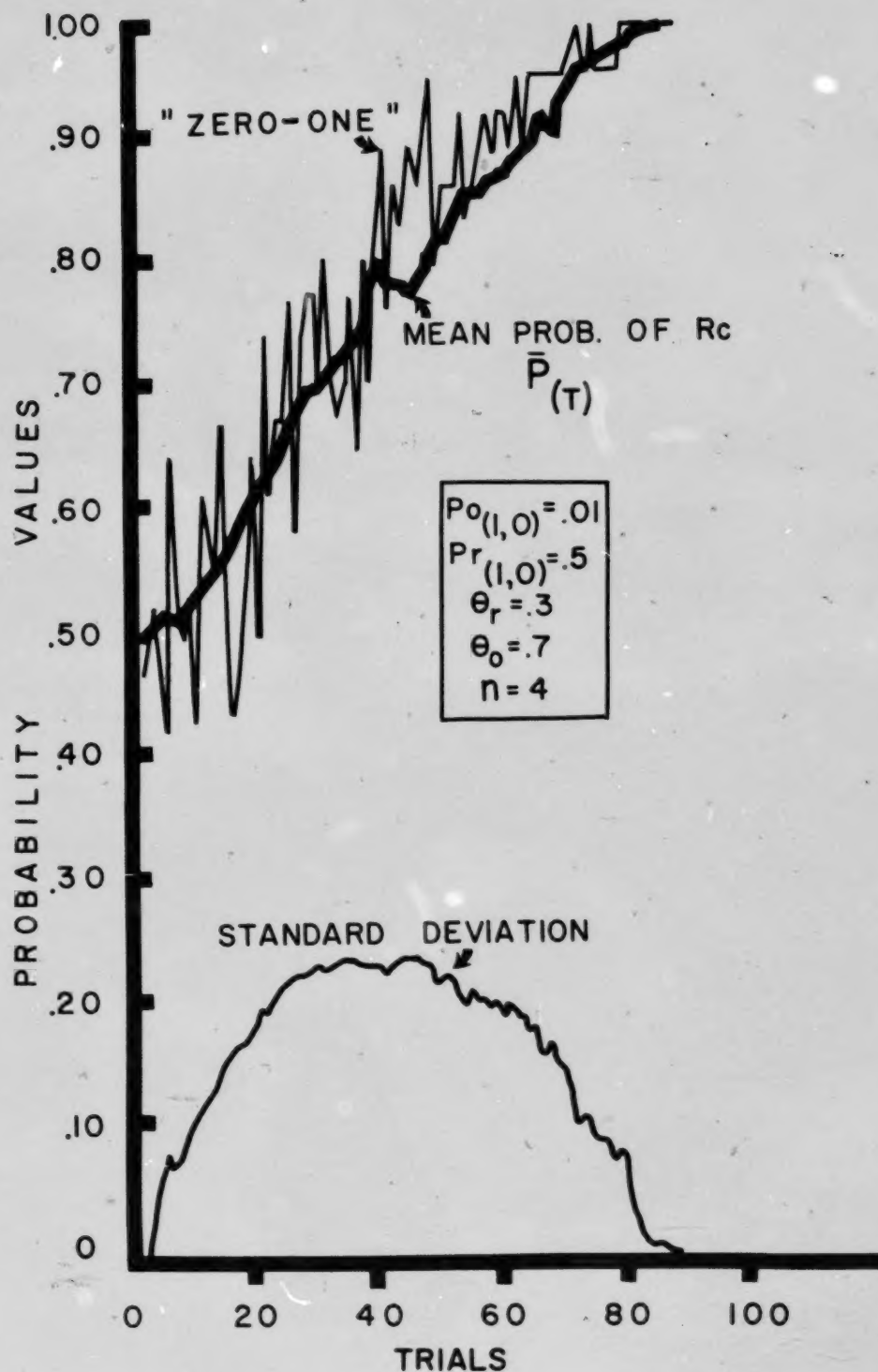


Figure 19. Three measures of Stat-child performance: mean probability, ($\bar{P}(T)$) of a group of 30; the standard deviation of $\bar{P}(T)$ values of the group; and the proportion of Stat-children making the correct choice ("Zero-one" function). A different group of 30 was used for making the Zero-one calculations.

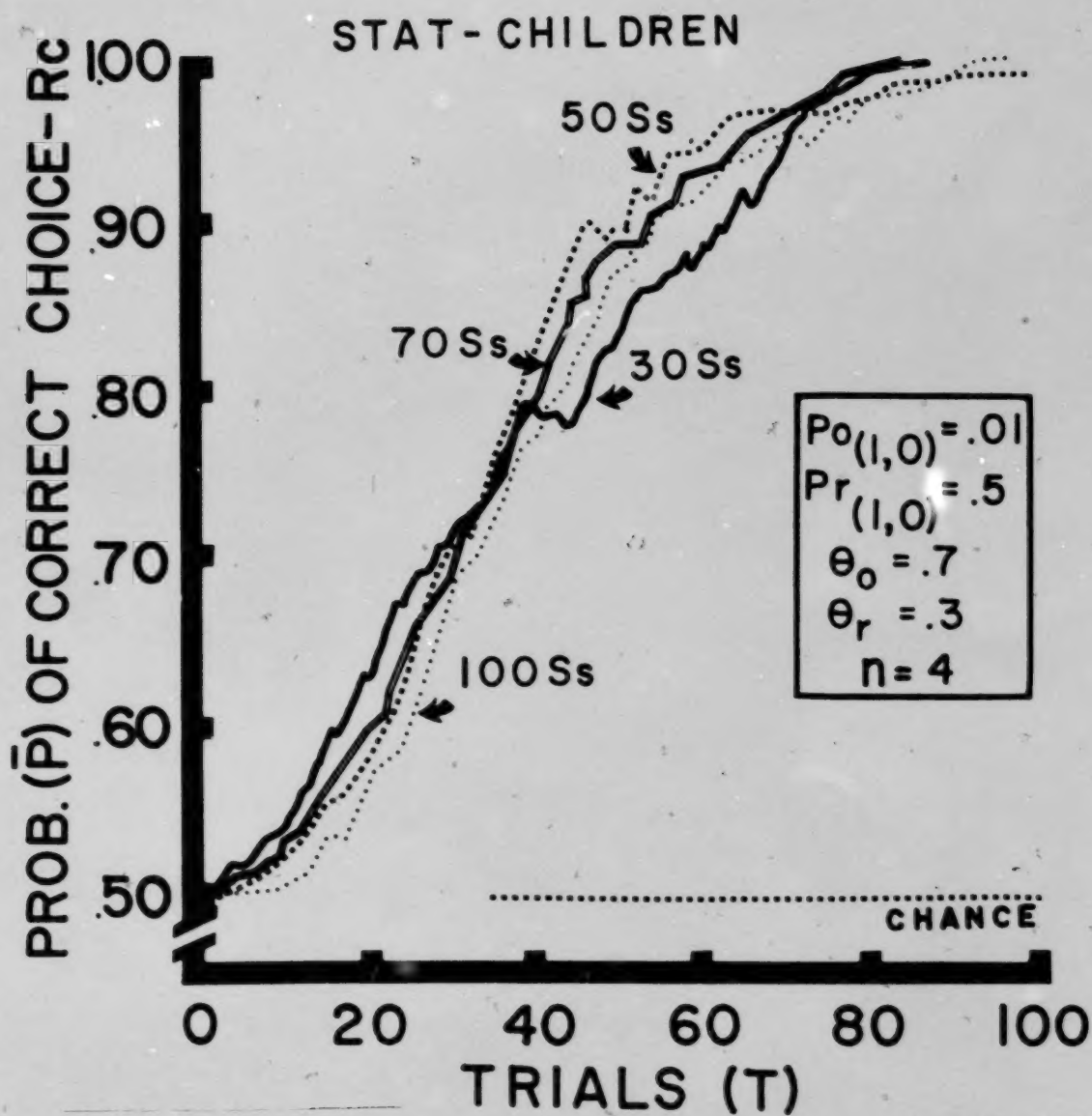


Figure 20. Mean P function: for groups 30, 50, 70, and 100 Stat-children under the parameter conditions indicated.

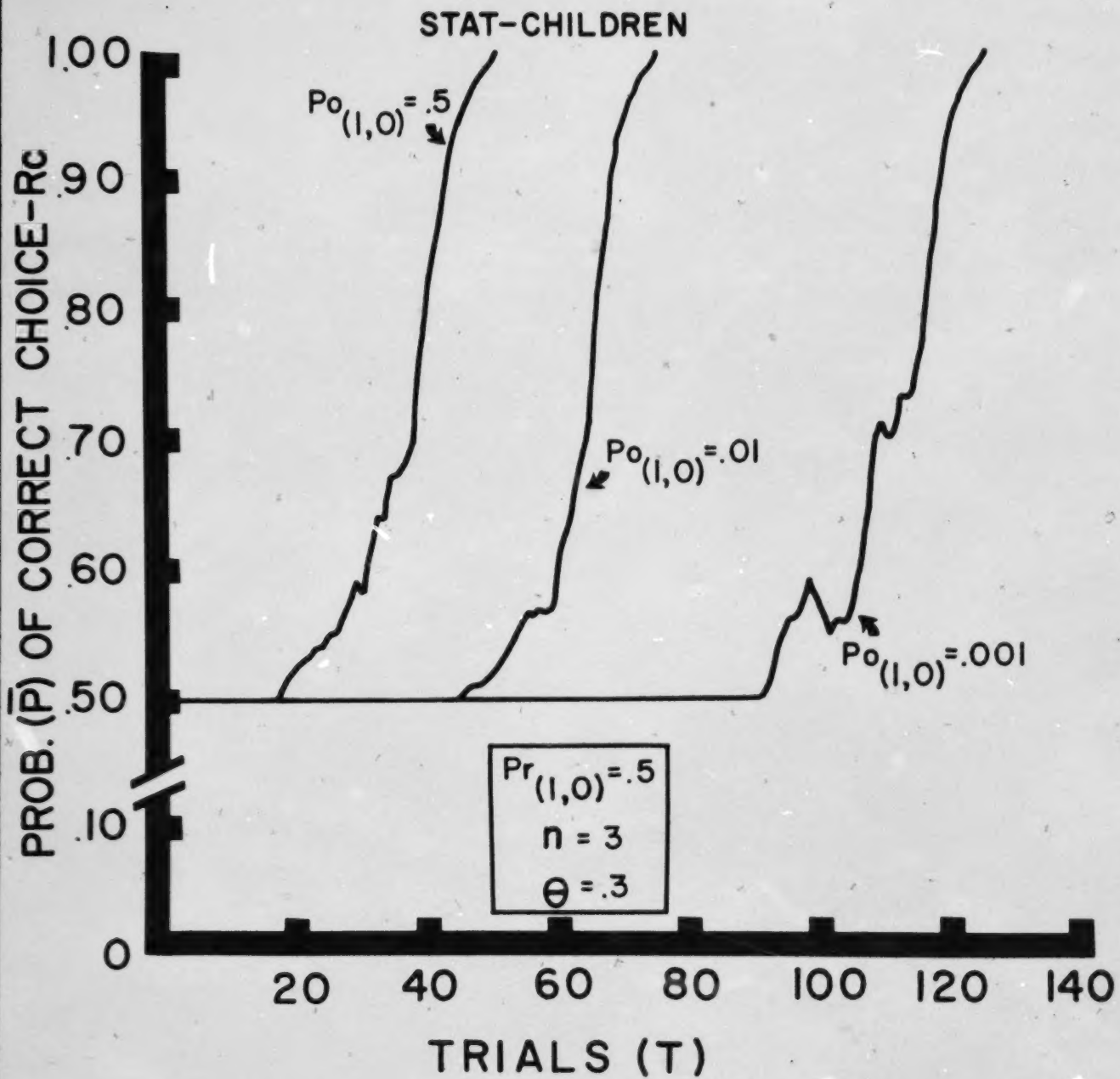


Figure 21. Backward discrimination learning curves for three groups of Stat-children varying in initial probability of attending to the relevant dimension.

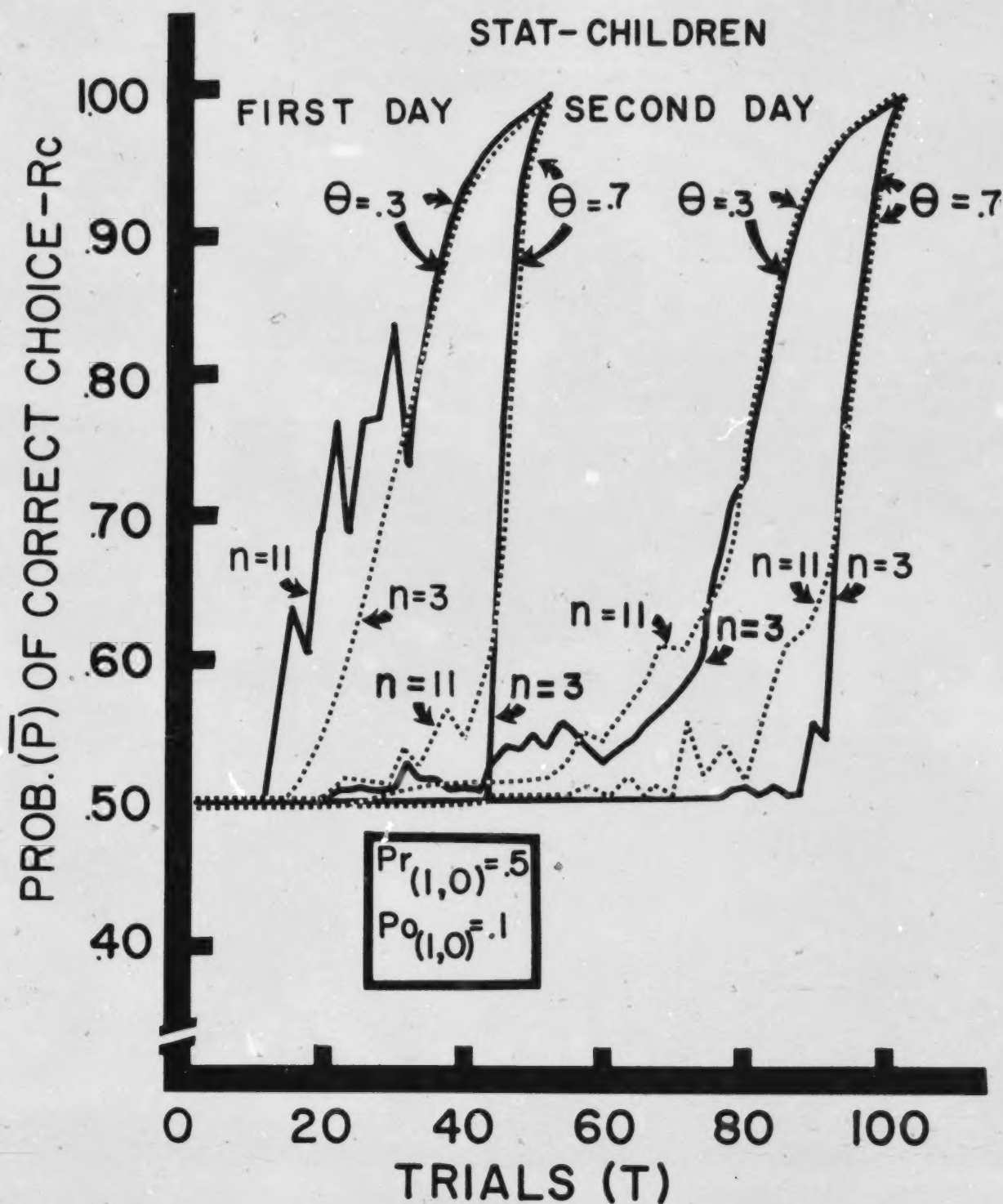


Figure 22. Joint effects of the parameters θ and n on mean performance of Stat-children reaching criterion on either the first or second day of training.

and $\theta = .3$ and $.7$. Reading the graph from left to right, the first pair of functions are backward curves for all the Stat-children reaching criterion of $P_{(T)} = .99$ on the first day with $\theta = .3$. Only a single Stat-child of a group of 20 under the $n = 11$ condition achieved criterion, and its $P_{(T)}$ function is plotted as the solid line. Reducing the number of dimensions to 3 causes an increase in number of Stat-children reaching criterion to 5. The backward curve of these 5 is shown next as the dotted line. The final portions of these functions overlap.

The next pair of curves differs from the first only in the value of θ , which has been changed to $.7$. A total of 6 subjects reach criterion on the first day under the $n = 11$ condition, while 15 achieve criterion under the $n = 3$ condition. These two functions differ only slightly, and not at all in the final portions; but both differ from the $\theta = .3$ functions in final rates of approach to asymptote.

The two pairs of functions at the right in Figure 22 represent the same parameter conditions as the first two pairs, but these are backward curves for Stat-children reaching criterion on the second day of training (Trials 26-50). The same conclusions can be drawn from these plots. For backward curves, the parameters $P_{(1,0)}$, and n effect lateral placement of backward functions but not the final slopes. The θ parameter, on the other hand, is directly related to final slopes. Other relations of parameters to backward learning curves will be presented when needed later in relation to data presentation.

Applications

Original Learning

The learning curves in Figures 3 and 4 of subjects separated on the basis of trials to criterion appear to differ in length of initial chance-level plateau rather than in slope of final approach to asymptote. This is taken as an encouraging sign that backward learning curves may be computed without doing violence to the forms of individual learning curves.

Differences in number of trials to criterion, such as those shown in Figures 3 and 4, could have been produced by temporary random differences in attention of the kind exhibited by groups of Stat-children with identical starting parameter values. However, the wide range of intelligence among these subjects makes it likely that individual parameter differences contributed to the results, with brighter subjects appearing among the fast learners and duller subjects affecting mostly the slow learning curves. If so, there is no evidence that subjects differed in rate of learning (θ); differences in $P_{(1,0)}$ and/or n could have produced the distribution of curves shown. More direct evidence can be obtained by comparing groups separated on the basis of intelligence as is done in the next section.

Intelligence

In Figure 5 it may be seen that subjects separated according to intelligence produce backward learning curves which differ in lateral position but not in final rate of approach to asymptote. While the effects of intelligence on performance level are statistically reliable (House and Zeaman, 1960a), statistics provide no standard method for deciding which of the parameters vary with intelligence. From the similar final rates we infer that there were negligible differences in θ among the subjects. Families of curves such as those shown in Figure 5 could be produced by variation in $P_{(1,0)}$ or n or some combination of the two factors. A bit of rational argument is needed to determine which is the most likely source of difference.

Suppose that our retardates tend initially to attend to certain "easy" dimensions, such as position, to the exclusion of almost all others. This would make $Po_{(1,0)}$ very low for color-form discriminations and account for the results. If, to pursue the contrary argument, we suppose that the parameter α is responsible for the results, we must then assume that our high MA group attends to fewer dimensions than the lower MA group. This assumption violates our intuitive notion that brighter subjects are able to discriminate more aspects of the surround than are duller subjects. We conclude that low MA subjects have lower initial probabilities of observing color and form than high MA subjects. Further consequences of assuming that subjects of low developmental level attend to fewer (and easier) dimensions are elaborated later in a general discussion of the relationships of theoretical parameters to retardation.

Stimulus Factors

If the trouble with retardate discrimination learning lies in their initial inattention to the stimulus dimensions we happen to choose as relevant, then logically the effects of intelligence could be minimized by selection of relevant stimuli having high initial attention-value. Position would certainly be a case in point, and it is true that retardates have little trouble learning position discriminations; in trainables with MAs in the 2-4 year range, position discriminations are learned with fewer than three errors on the average (House and Zeaman, 1959). Multidimensional object discrimination represents another instance, and the fast learning of this problem (Figure 6) shows "junk" to have high attention-value also.

In general, the discrimination difficulty scale, for combinations of color and form, object and pattern, established in Figure 6 is interpreted to reflect differences in $Po_{(1,0)}$ for the various dimensions. It could, of course, be true that dimensions not likely to be noticed might also be those hard to learn, in which case easy dimensions would have not only high Po s but high α s as well. The shapes of the functions in Figure 6 weigh against this possibility, since, regardless of difficulty, the final rates of approach to asymptote (controlled mostly by θ) do not vary appreciably.

Other stimulus factors will be considered in a later section.

Transfer Operations

The "attention hypothesis" (that retardates suffer from low initial probability of observing certain relevant dimensions rather than from poor ability to learn which of two observed cues is correct) is to us a hopeful and exciting notion because it leads so quickly to a search for experimental operations that may change $Po_{(1,0)}$ and hence accelerate discriminations of subjects who might otherwise seem to be natively and irreversibly poor discriminators.

Several transfer operations have been discovered that do control $Po_{(1,0)}$. Perhaps the most dramatic is Object-to-Pattern transfer (House and Zeaman, 1960b). Color-Form Pattern discriminations are difficult, as we see in Figure 6, while Color-Form Object discriminations are easy. Using the easy object discrimination to focus attention on the relevant color-form dimensions, we found that attention would then transfer strongly to the difficult pattern problem.

Object-to-Pattern transfer. - Three matched groups of retardates (MA 2-6 years) were employed in this experiment: the P-1 Group was trained to discriminate between two-dimensional patterns such as a red cross and blue triangle; the O-1:P-1 Group learned two problems, first a color-form object discrimination (O-1) such as a three-dimensional red cross vs. a three-dimensional blue triangle, and second a pattern discrimination (P-1) with the same positive and negative color-form cues (a flat red cross vs. a flat blue triangle); Group O-2:P-1 also learned two problems, an object discrimination

followed by a pattern, except that in this case the positive and negative cues of the object discrimination were different colors and forms than those of the pattern discrimination (e.g., from green square vs. black circle in the object discrimination shifted to red cross vs. blue triangle in the pattern.)

Figure 23 presents the results of this experiment in the form of backward learning curves for the three groups. Of the 19 subjects in the P-1 Group,

Insert Fig. 23 about here

only 4 learned within the 500 trials allowed, and their backward learning curve is shown at the far right. In the O-1:P-1 Group, 13 of the 19 subjects achieved criterion on the original object discrimination (O-1), and then were placed on the corresponding pattern discrimination (P-1). All 13 learned the pattern with the backward learning curve shown at the far left.

In the O-2:P-1 Group, 14 of the 22 subjects learned the object discrimination (O-2) and were then shifted to the pattern problem (P-1). Of this group of 14 subjects, 10 learned the pattern problem. Their backward learning curve is the middlemost function in Figure 23.

What do these findings mean theoretically? In the O-1:P-1 condition, the objects and patterns not only have the same relevant color-form dimension (making $Po_{(1,0)}$ high for subjects as they enter P-1), but the specific cues of the relevant color-form dimension are also largely the same for object and pattern (making $Pr_{(1,0)}$ high also for the second problem). With transfer of both instrumental and observing responses, there is little learning left to do under the pattern discrimination of this group to bring P to criterion. (See equation 1)

In the O-2:P-1 condition we would expect transfer of the relevant observing response but not the correct instrumental response (R_1). This follows from the dimensional nature of attention and the greater specificity of approach-avoidance tendencies. Changing the specific color-form characteristics but keeping color-form relevant allows $Po_{(1,0)}$ to remain high, while $Pr_{(1,0)}$ goes to the chance level of $\frac{1}{2}$ as the subject faces new positive and negative cues to which he has not been previously trained to approach or avoid. Equation 1 tells us that this group must start at a chance level of performance (because $Pr_{(1,0)} = .5$) and Figure 9, 10, and 11 show that the high value of $Po_{(1,0)}$ will tend to make for rapid improvement in performance.

The relative locations and shapes of the curves in Figure 23 are theoretically accountable allowing variations of $Po_{(1,0)}$ and $Pr_{(1,0)}$ in the predicted directions. No precise parameter estimations have been made for the data, but even these semi-quantitative predictions are better than those which follow from other discrimination theories.

From both the theoretical and applied points of view a dramatic point of this experiment is this: a discrimination that could not be taught by prolonged differential reinforcement could be taught by harnessing some of the dynamics of attention.

Object-to-Pattern transfer is a special case of a more general class of experimental operations that achieve high starting probabilities of attention to the relevant dimension - the class of intradimensional shifts. Other instances of intradimensional shifts will be described presently.

Reversal. - Another more usual method of achieving high $Po_{(1,0)}$ is that of discrimination reversal, or switching the roles of the positive and negative cues. Associating reinforcement with the previously negative stimulus object does two things theoretically: it leaves $Po_{(1,0)}$ undisturbed, and high, if the pre-reversal learning nears completion; but it reduces the instrumental response probability $Pr_{(1,0)}$ to the complement of its pre-reversal level, that is, to a value approaching zero after a well-learned original discrimination.

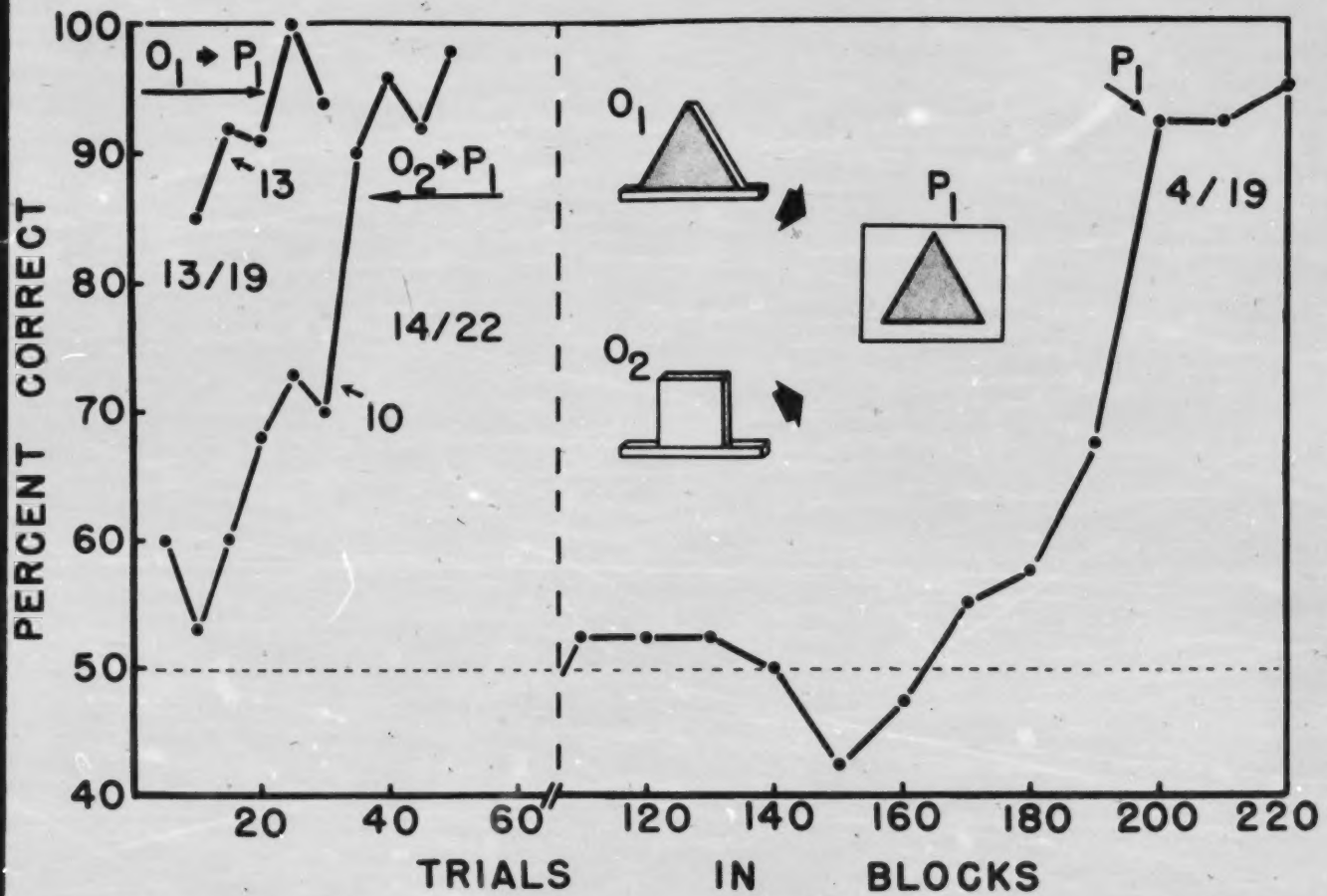


Figure 23. Object-to-pattern transfer. Backward learning curves of a Pattern Group (far right) and two Object-to-Pattern Transfer Groups (at the left) show large transfer effects attributable to attention.

Thus, there should result a positive transfer tendency to the reversal problem from the carry-over of a high $Po_{(1,0)}$ and a negative transfer tendency from the low $Pr_{(1,0)}$. The net result or balance of the two effects is dependent upon specific parameter values, chiefly θ_0 and θ_r . The system is therefore capable of predicting either an overall positive, negative or zero transfer on the reversal problem, assuming different rate constants. From this it can be seen that until we arrive at the stage of borrowing parameters fixed in earlier experimentation, no critical test of attention theory can be made by comparing speed of reversal learning with that of original learning or other transfer conditions.

Fortunately, among the many possible predictions made by the theory about reversal learning, there are some unusual ones made about the shapes of reversal curves. Certain combinations of parameters yield reversal functions featuring a midplateau as shown in Figures 16 and 18. It is sufficient for our present purposes to demonstrate that such unusual curve shapes do occur under some experimental conditions of reversal learning.

Figure 24 is a forward learning curve for the reversal of a position discrimination for a group of trainables (MA 2-6). A section of positive

Insert Fig. 24 about here

acceleration in the middle of this function is clear. The criterion of learning required by this particular study (Zeaman and House, 1962a) was less stringent than usual - six successive correct responses. Data are available, however, from a position reversal problem in which a more severe criterion was adopted (House and Zeaman, 1959) - ten successive correct responses, and the reversal learning curve for this condition is shown in Figure 25.

Insert Fig. 25 about here

Note that with the more stringent criterion the positive acceleration is less apparent.

Having shown that the criterion stringency exercises some control over the midplateau, we ask whether this control can be explained theoretically. Figure 26 shows that that it can. Plotted are theoretical reversal functions

Insert Fig. 26 about here

with fixed parameters $\theta = .7$, $n = 5$, $Pr_{(1,0)} = 0$; the parameter $Po_{(1,0)}$ is varied with values .99 for the upper curves and .5 for the lower. Solid lines are means of 20 Stat-children; dotted lines represent the expected operator approximation. Reducing the value of $Po_{(1,0)}$ makes the midplateau more pronounced. This effect is discernible in both the expected operator and the Stat-child plots, even though there are sizable discrepancies in magnitude between the two approximations.

A step in the logic has been omitted. To complete the theoretical analysis of the differences between Figures 25 and 26, we need to say why it is reasonable to assume that differences in the learning criterion should produce differences in $Po_{(1,0)}$. The reasoning is not involved. A more stringent criterion for original learning tends to make both the relevant observing response and the correct instrumental responses stronger at the conclusion of the original learning problem. But $Pr_{(1,T)}$ approaches unity faster than

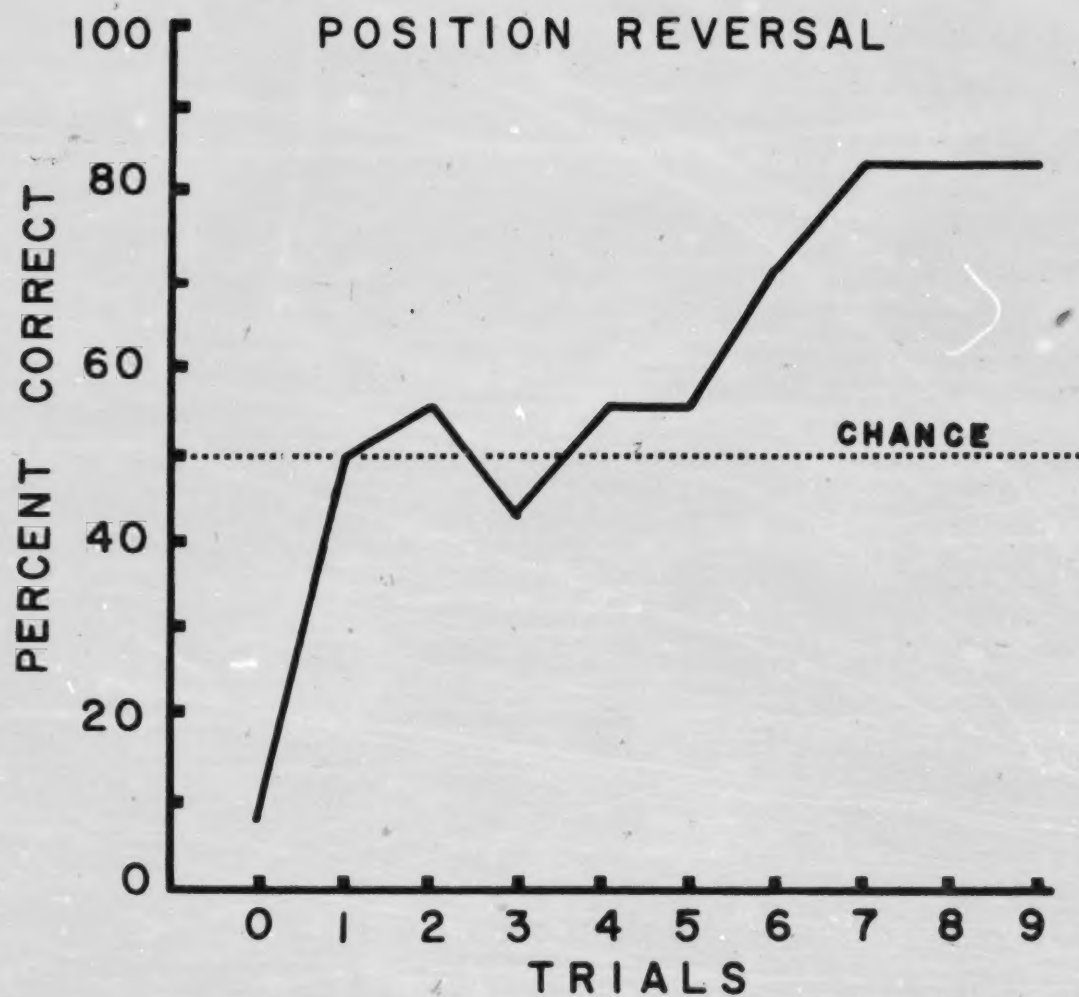


Figure 24. Position reversal data for a group of 24 retardates.

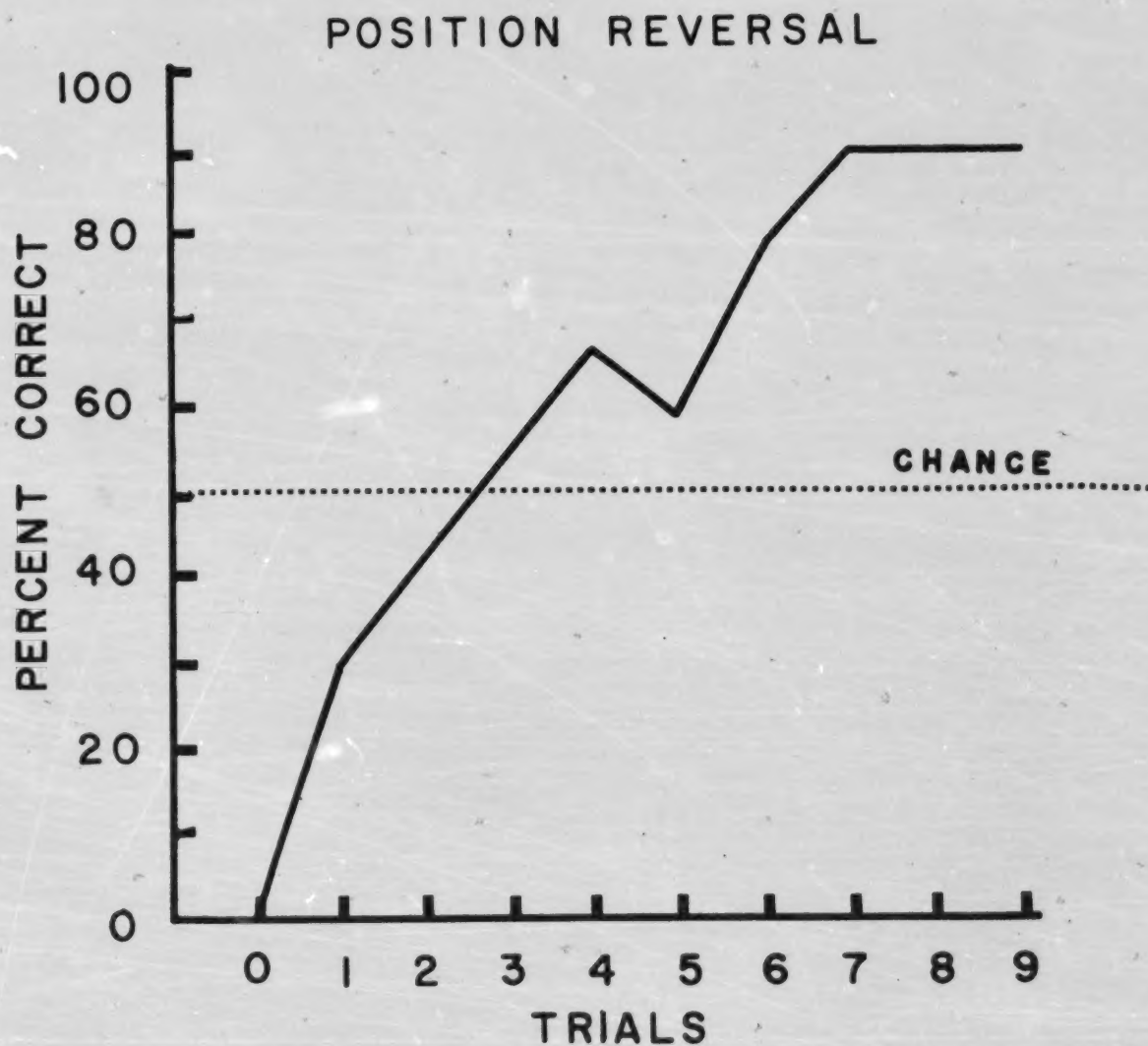


Figure 25. Position reversal data for a group of 16 trainable retardates required to reach a more stringent criterion of original learning than the subjects whose data is plotted in Figure 24.

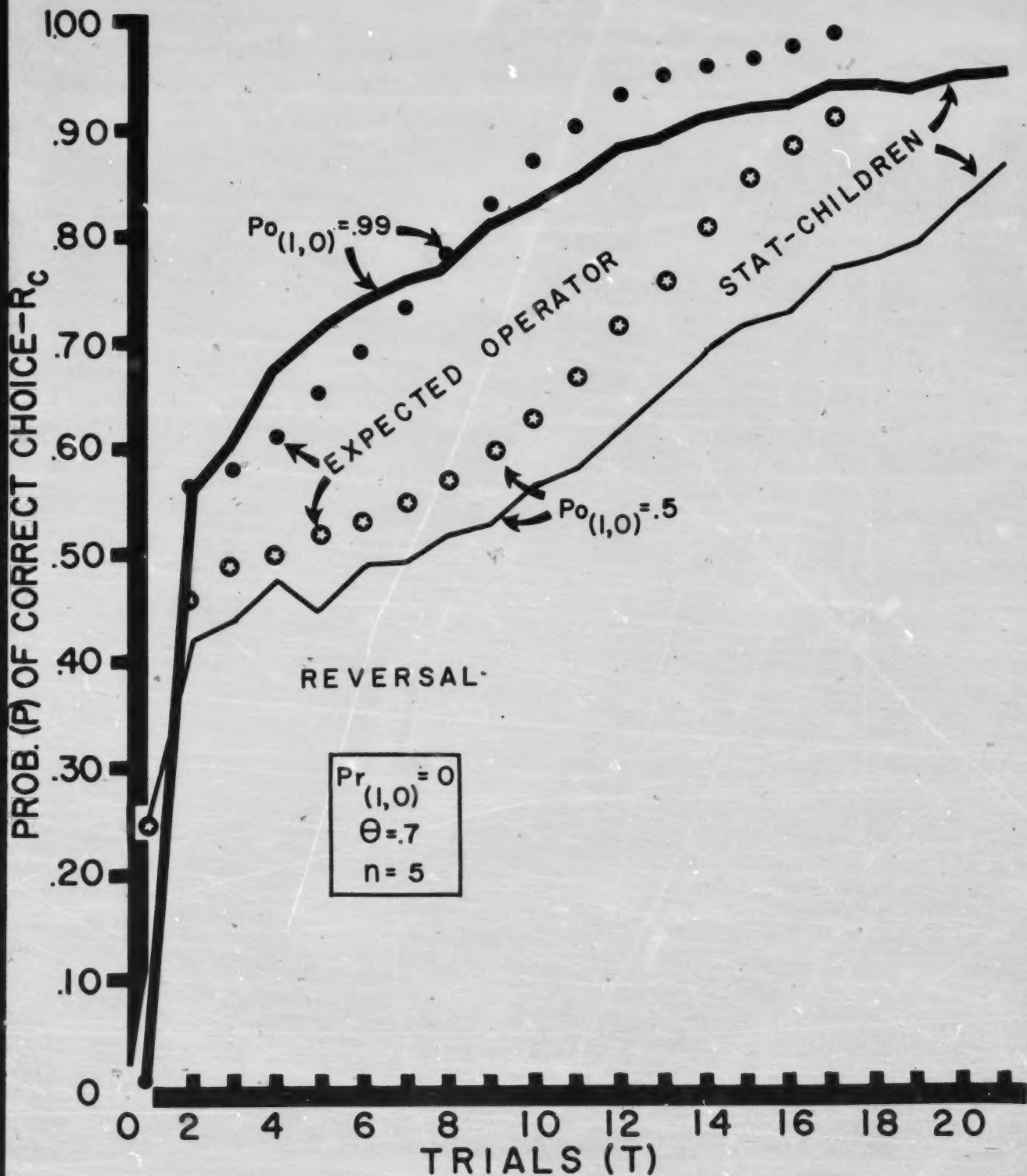


Figure 26. Effects of different levels of original learning on theoretical reversal functions. Starting reversal with a well-learned observing response ($P_{O(1,0)} = .99$) tends to produce less of a midplateau than starting with a relatively weak observing response ($P_{O(1,0)} = .5$).

$Po_{(1,T)}$ for all cases of equal θ_s and $n > 2$. For this reason, overlearning trials will likely be acting to bring $Po_{(1,T)}$ toward unity after $Pr_{(1,T)} = 1.0$. Therefore, a stringent criterion will tend to make $Po_{(1,0)}$ closer to 1 to start reversal, while a weaker criterion makes $Po_{(1,0)} < 1.0$.

No systematic attempt has yet been made to find the best parameter combinations for these empirical reversal functions. If equal θ_s are assumed, then $\theta = .7$ is the right order of magnitude for n values near 5. However, to achieve the degree of positive acceleration exhibited by Figure 25 would require unequal θ_s with $\theta_o > \theta_r$.

Another indication that reversal midplateaus occur under other experimental conditions is given in Figure 27. In this study (Clack and Zeaman, 1960),

Insert Fig. 27 about here

original learning and reversal problems are solved by a group of retardates and a group of normal children matched for MA (2-4 years). Random schedules are employed in this position discrimination experiment. Candy rewards are available on 75% of the trials in one position and 25% in the other. During reversal, the position previously receiving 75% reinforcement now gets 25%. The midplateau is strikingly clear in the reversal function of the normal group. Positive acceleration is also present in the retardate curve, but the initial sharp rise from Trial 0 of reversal is obscured by retention loss during a month's gap in training for this group.

We have not yet presented the submodel of the theory which handles schedules of reinforcement, but Figure 27 clearly sets a requirement for such a model. It must predict reversal midplateaus for some parameter combinations. A discussion of this point is left for a later section.

Intradimensional and extradimensional shifts.— Two transfer operations providing strong but differential control of initial observing response probabilities are intradimensional and extradimensional shifts. For an intradimensional shift, the subject is given, after original learning, a new pair of relevant cues varying along the same dimension as the cues relevant during original learning. Repeating an example given earlier, if blue and yellow were the relevant cues during training, red and green might be the new cues during intra-dimensional color shift. If, as we have assumed, attention operates on entire dimensions, there should be no loss in probability of observing the new cues. The instrumental approach probabilities, however, go to chance level since the new positive stimulus is designated randomly. Consequently, at the start of intradimensional shift, $Po_{(1,0)}$ should be high, and $Pr_{(1,0)}$ at .5. Most of the learning to be done after intradimensional shift is, therefore, instrumental.

In contrast, the operation of extradimensional shift changes a previously irrelevant dimension to relevant. From color relevant, we might shift to form. The result is, of course, a low $Po_{(1,0)}$ and a chance level $Pr_{(1,0)}$. Table 2 summarizes the conditions of starting probabilities for five different

Insert Table 2 about here

problems. The question marks in the Table designate probabilities that vary from problem to problem. "High" and "low" approximate unity and zero, respectively, depending upon the performance criterion adopted for pre-transfer learning. The Oddity Problem is discussed later.

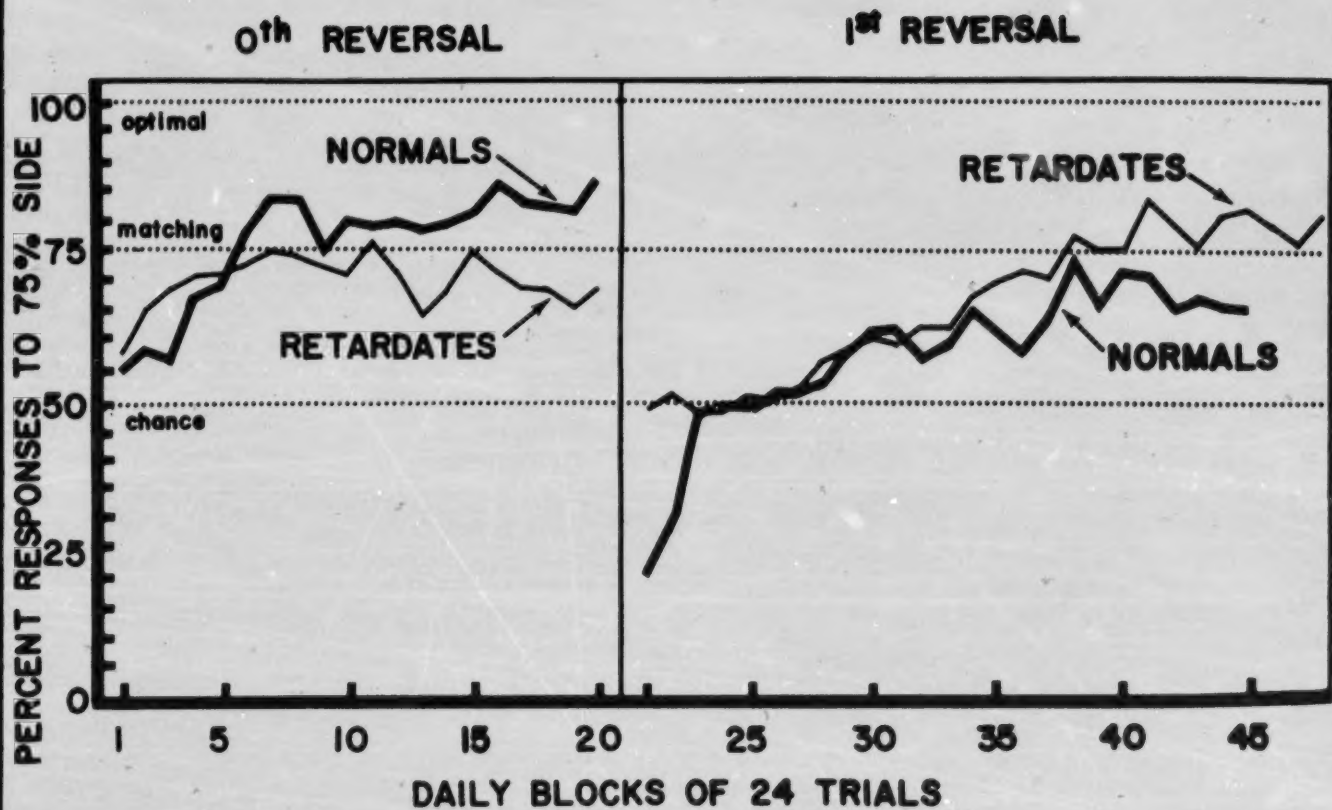


Figure 27. Original learning and reversal of a position discrimination problem with 75% (random) reinforcement of the correct response and 25% reinforcement of the incorrect response. Both retarded and normal subjects were run. Performance is measured as the group mean percentage choice of the more reinforced position. Of interest is the evidence of a midplateau in the reversal functions.

Table 2
Starting Probabilities of O_1 and R_1

Problems	Starting Probabilities	
	Relevant Observing Response ($Po_{(1,0)}$)	Correct Instrumental Response ($Pr_{(1,0)}$)
Original Learning	?	.5
Reversal	high	low
Intradimensional Shift	high	.5
Extradimensional Shift	low	.5
Oddity	?	high

A study has been done comparing performances of matched groups of retardates on Intradimensional Shift, Extradimensional Shift, and Reversal Problems (House and Zeaman, 1962a) following a high degree of overlearning. From Table 2, some predictions can be made about the relative difficulty of these three problems. If starting probabilities are the only factors differentiating the problems (our working hypothesis), both extradimensional shift and reversal problems should be more difficult than intradimensional shift. Strictly speaking, other comparisons are indeterminate without a knowledge of parameter values. However, with the high degree of overlearning used in the experiment, reversal should be facilitated and extradimensional shift retarded. A reasonable guess, then, is that the problems should rank themselves in order of increasing difficulty: intradimensional shift, reversal, and extradimensional shift.

The expected shapes of the individual discrimination learning curves can also be deduced for the three conditions. Both the intradimensional shift and reversal curves should begin to rise immediately with negative acceleration; the extradimensional shift curve should have an initial tendency to remain at chance followed by a period of positive acceleration.

In the experiment we have done comparing performance on these three problems (House and Zeaman, 1962a), three matched groups in the MA 6-8 range were given original training in discrimination of objects differing in only one dimension, either color or form. If form were relevant, then two values of the irrelevant color dimension were varied every trial in random association with the positive and negative cues. This means that the positive stimulus might be either a green square or a red square, for instance, and the negative stimulus either a red triangle or a green triangle.

Following an original learning problem with form relevant for some subjects and color relevant for others, the three groups were transferred to a Reversal, an Intradimensional Shift, or an Extradimensional Shift condition. Table 3 illustrates a set of possible stimulus arrangements for the first

Insert Table 3 about here

three trials of the four problems for subjects learning originally a form-relevant discrimination.

The data from this experiment are presented in Figure 28 in the form of backward learning curves. The median subject in both the Intradimensional

Insert Fig. 28 about here

Shift and Reversal conditions reached criterion within the first day's allotment of 25 trials; while the median subject of the Extradimensional Shift Group took three days to learn. Trials have been blocked in fives to achieve enough stability to see the major trends in the data. Differences among the three conditions are statistically significant, and in the expected direction. The forms of the functions also agree roughly with theoretical predictions. Figure 29 graphs the performances of three groups of Stat-children with para-

Insert Fig. 29 about here

eters appropriate for the three experimental conditions. There are not a lot of completely free parameters here. Since all the subjects were brought

Table 3

A Sample of Stimulus Arrangements for Original Learning
And Three Transfer Problems on Several Trials

Trial No.	Original Learning	Reversal	Intradimensional Shift	Extradimensional Shift
1.	(+) (-) Green Red Square Circle	(+) (-) Green Red Circle Square	(+) (-) Red Green Triangle T	(-) (+) Green Red Square Circle
2.	(-) (+) Green Red Circle Square	(-) (+) Red Green Square Circle	(+) (-) Green Red Triangle T	(+) (-) Red Green Square Circle
3.	(-) (+) Red Green Circle Square	(-) (+) Green Red Square Circle	(-) (+) Red Green T Triangle	(-) (+) Green Red Circle Square

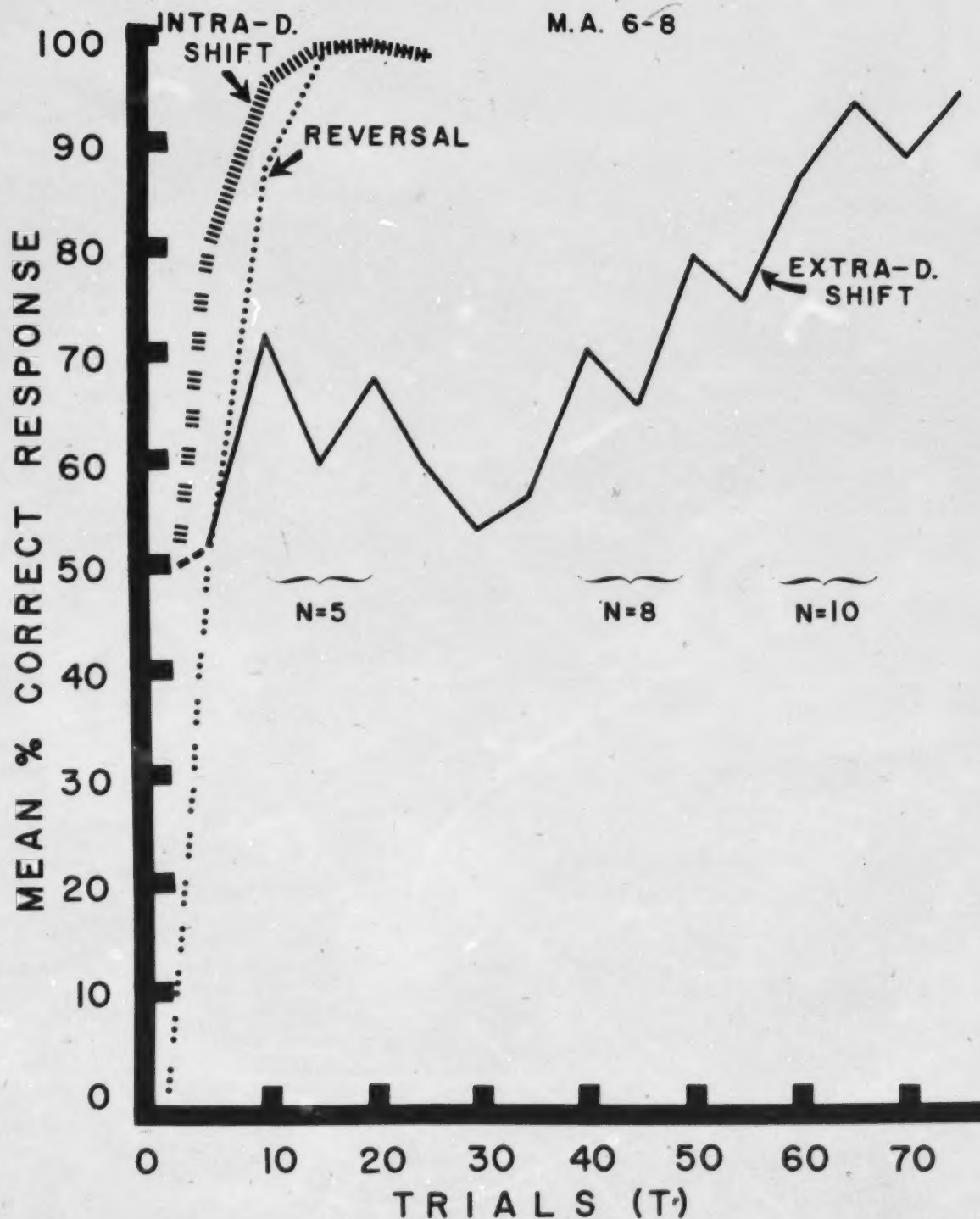


Figure 28. Backward learning curves for three groups of retardates in the Intradimensional Shift, Reversal, and Extradimensional Shift conditions. Mean percentages of correct choices are plotted for blocks of five trials. Broken lines connect the initial block with the starting level for each condition.

The number of subjects contributing data to various sections of the Extradimensional Shift Group function diminishes from 10 to 5.

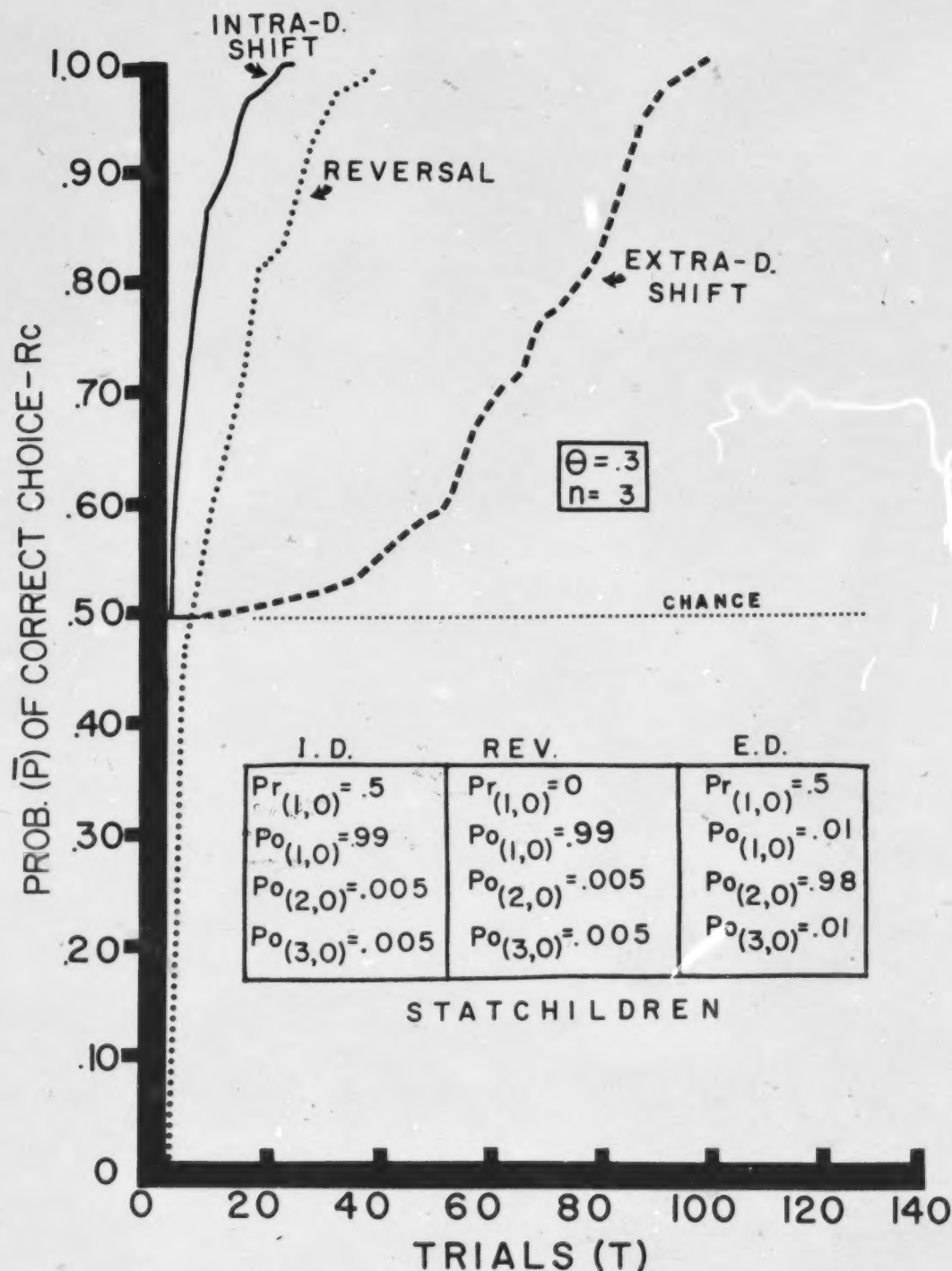


Figure 29. Theoretical curves for Intradimensional Shift, Reversal, and Extradimensional Shift Problems.

Performances of three groups of 20 Stat-children are plotted as backward learning curves except for the first two points of each function which come from forward learning curve calculations. The lateral positioning of the curves is that of the median subject in each group. Parameter values common to all three curves are indicated as are the distinctive parameters for each condition.

to a strong criterion of original learning, the starting probabilities are restricted in their range. $Po_{(1,0)}$ must approach unity for the Intradimensional Shift and Reversal conditions, which implies that $Po_{(1,0)}$ must be near zero for the Extradimensional Shift Group. The rules of experimentation fix $Pr_{(1,0)}$ at .5 for the Intra- and Extradimensional Shifts, and at zero for Reversal (given a $\bar{p}$ near one at the end of original learning). The irrelevant observing response probabilities are both necessarily very small at the start of Reversal and Intradimensional Shift, whereas one of the irrelevant observing response probabilities must be very high in the Extradimensional Shift condition. In Figure 29 we have used $Po_{(2,0)} = .98$ and $Po_{(3,0)} = Po_{(1,0)} = .01$. This was a slight mistake; we should have had $Po_{(2,0)} = .99$ and the other two probabilities at .005 to be consistent with the other two curves. Reducing $Po_{(1,0)}$ to .005 might have shifted the Extradimensional Shift curve to the right.

There is nothing in the data to tell us how many dimensions are operating, nor are there yet any strong rational grounds for fixing the range of the parameter n , so we have tried out a number of different values of n in combination with values of θ . Three dimensions and $\theta = .3$ give a rough approximation of the data as may be seen by comparing Figures 28 and 29. We have shown earlier that the slopes of the later portions of the discrimination learning curves are largely controlled by the θ parameter. The slopes displayed in Figure 28 are roughly accommodated by θ s of the magnitude of .3. This may be an underestimation of θ since methods for obtaining empirical backward curves result in less steep functions than methods used with Stat-children. This difference arises from the necessity for measuring criterion performance over blocks of trials for empirical data.

The correspondence of theory and data given by a comparison of Figures 28 and 29 is encouraging. The points of largest discrepancy occur in the Extradimensional Shift curve, especially in the early portions. The few unusually high points occurring here may be random fluctuations because the points are based on relatively few measurements. Although ten subjects contributed to the Extradimensional curve, only five had data extending backwards three days (75 trials). As a result, the first five points include data from only five subjects.

Figure 30 displays the expected operator curves with the same parameter

Insert Fig. 30 about here

combinations as those of the Stat-children in Figure 29. While the two sets of functions in Figures 29 and 30 are by no means identical, the formal and quantitative similarities are great enough to accept each as an approximation of the other for the particular parameters readed here.

A question of interest not answered by Figure 28 is the relation of original learning and Extradimensional Shift. It will be recalled that some of the subjects in the Extra-dimensional condition are transferred to a form and some to a color discrimination. The original learning of both these single cue problems is slower than that of the Extradimensional Shift conditions. While intuitively the Extradimensional Shift may seem to be a negative transfer condition, we have shown earlier that such an intuition does not necessarily agree with the theory. Having a single, strong, competing irrelevant observing response may be theoretically facilitating (cf. Figure 14). If, in fact, the two irrelevant observing responses were of equal strength at .495 instead of having one strong cue at .98 and one at .01 for the Extradimensional Shift Stat-children (Figure 29), the Extradimensional curve would have been shifted 25 trials to the right and would have had a longer chance portion.

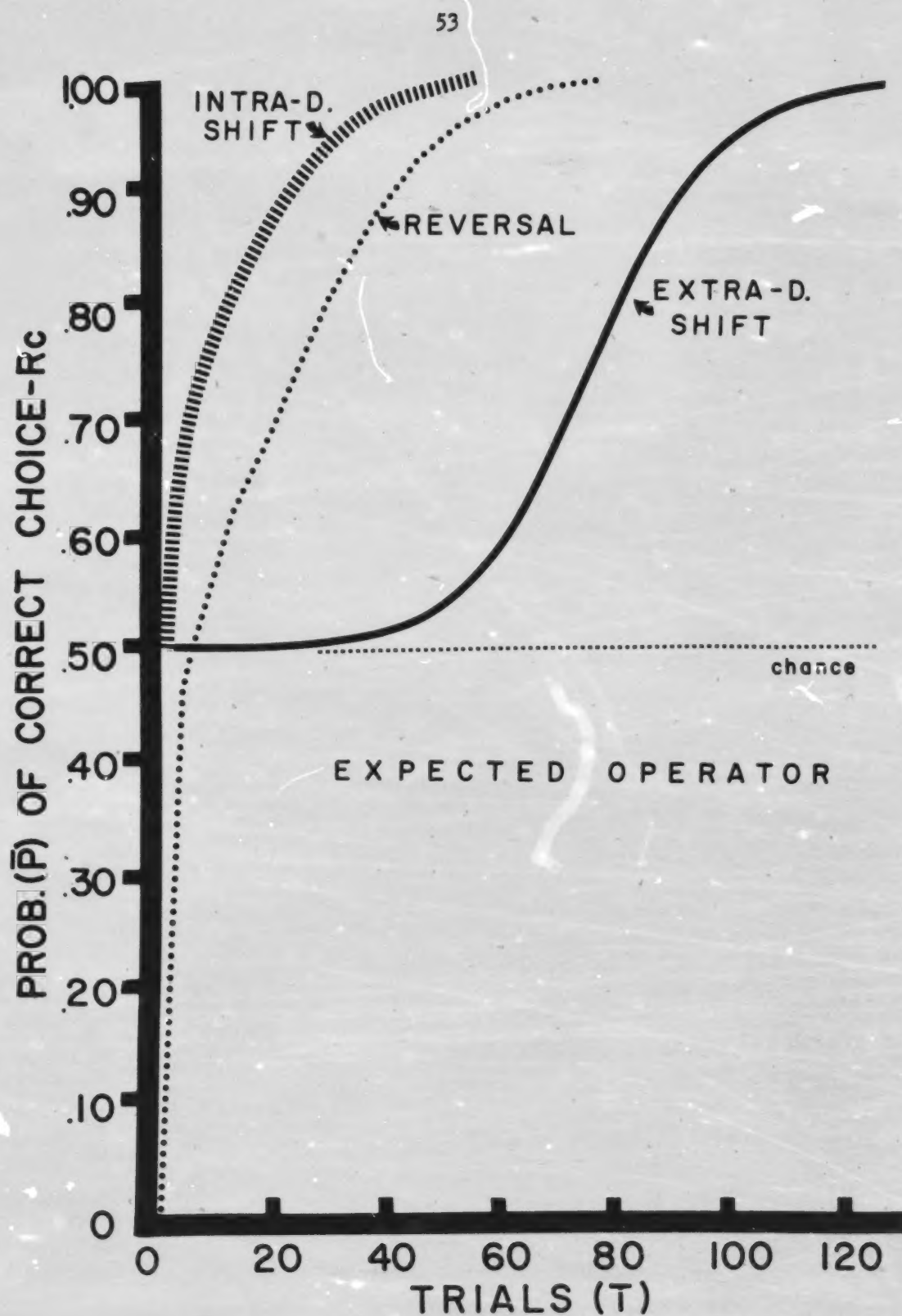


Figure 30. Expected operator approximations for the three transfer conditions. Parameter combinations are the same as those of Figure 29.

The superiority of intradimensional shifts over extradimensional shift has been demonstrated in variants of the present design by Kurtz (1955) and Eckstrand and Wickens (1954) with college students, and by Bensberg (1958) with retardates. In summary, it seems conservative to remark that Attention theory can handle many aspects of the data from these three transfer operations, intradimensional shift, reversal and extradimensional shift, as well as from original learning. And no obvious disconfirming evidence has turned up.

Other Factors Controlling Attention

Investigation of stimulus variables of greater complexity than those previously mentioned have led us to infer that some of these factors may also exert control of initial attention - P_o (1,0). Stimulus novelty is one striking factor.

Novelty. - It has been shown in an illustrative experiment that the introduction of one novel stimulus in place of one of the familiar stimuli can bring about sudden and unexpected learning of discriminations apparently too difficult for some of our subjects (Zeaman, House, and Orlando, 1958). These results were not attributable to simple preference for or aversion to novel or familiar stimuli since both novel positive and novel negative cues produced facilitation. Novelty or familiarity appear to be discriminable aspects of stimuli. Theoretically, novelty enhances P_o (1,0).

This is not to say that novelty can not also be a preferred aspect in some contexts, and thus raise Pr (1,0) above chance. Such an effect appeared unexpectedly in an experiment designed to discover the relative effects of reward and nonreward in the discrimination learning of the moderately retarded (Zeaman & House, 1958b). A technique was employed of presenting a single stimulus, either positive or negative, on the first trial of a junk-object discrimination problem. On the second and subsequent trials both positive and negative stimuli appeared in the standard two-choice, simultaneous procedure. After being forced on the first trial to choose the negative stimulus without reinforcement, subjects then strongly preferred, on the second trial, the positive (and novel) stimulus. For the converse operation, after an initial forced choice of the positive stimulus with reinforcement, the children preferred the negative (and novel) stimulus on the following trial. The results appear in Figure 31. Each of 14 subjects was tested with the above procedures on 30

Insert Fig. 31 about here

different junk discrimination problems for each of the two conditions: (a) Positive (as labelled in Figure 31), i.e., forced initially to choose the single positive stimulus with reinforcement, and (b) Negative, i.e., forced to the negative stimulus without reinforcement. The results are most easily explained as a tendency on Trial 2 to approach the novel stimulus, regardless of whether the familiar stimulus had been rewarded or not rewarded on the first trial. Similar findings of attraction to a novel stimulus have been reported in primate subjects (e.g. Harlow and Hicks, 1957).

The conclusion presented with our introduction of this assemblage of data may be repeated: novelty is not only a discriminable aspect of stimuli exercising strong control over attention (P_o), but it also may be a preferred aspect, theoretically acting on Pr .

The effects of novelty present us with a paradox, resolvable only by assuming attention to be a central process. The reasoning is straightforward. A stimulus cannot be judged novel except in relation to a prior series or background of nonnovel (familiar) stimuli. But such a judgment must require that the subject have some trace, record, or engram of previous (familiar)

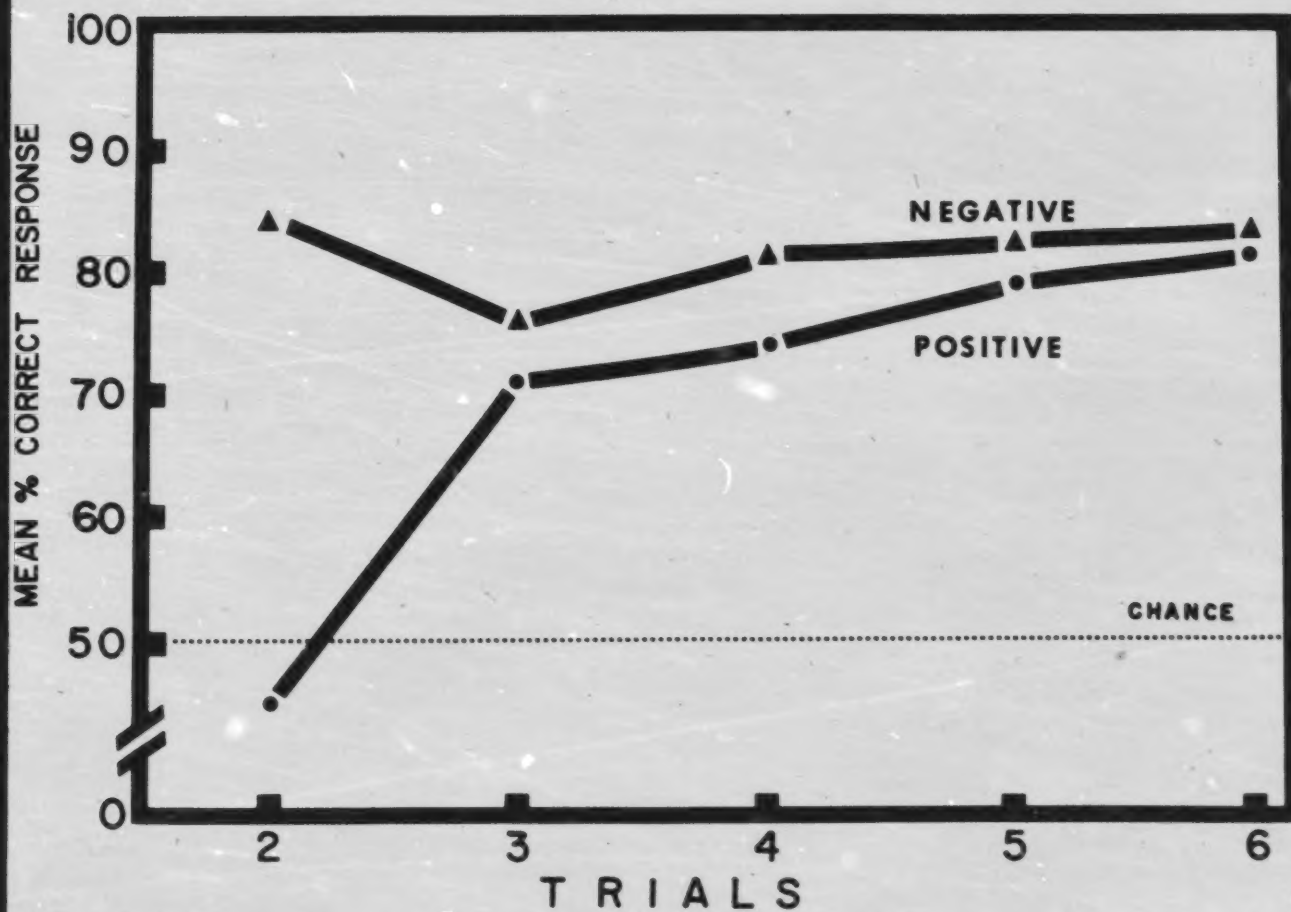


Figure 31. Discrimination learning curves for the Positive and Negative conditions. The funnel-shape of the two functions is interpreted as the result of a tendency to attend to and approach novel stimuli.

stimuli to contrast with the present novel cue. If seeing novelty requires some record of previous stimulation, then the previous stimulation must have gotten past at least the peripheral stages of sensory reception. We are forced to the conclusion that subjects who fail discriminations because of inattention are nevertheless storing stimulus information somewhere. Attention is, therefore, a central process.

Reward as a cue controlling attention. - Candy has high attention value for our subjects, but cannot act as a cue since it follows rather than precedes discriminative behavior. Shepp (1960) has presented evidence that anticipatory responses - candy expectations - may precede choice and provide distinctive cue feedback.

In one experiment Shepp used two kinds of candy - a miniature marshmallow and a Hershey chocolate Kiss - with trainable subjects (MA 2-4). One group (Successive -D) of 16 subjects learned a successive junk discrimination with one kind of candy always on the left and the other always on the right. In a typical problem, if two identical soap dishes were present, the subjects would find a marshmallow on the left, nothing on the right; if two pot covers were present, the subject would find a Hershey Kiss on the right, nothing on the left. As shown in Figure 32, this group learned considerably faster than the

Insert Fig. 32 about here

Successive -R Group (16 subjects) which had marshmallows and Hershey Kisses placed randomly left or right. According to Shepp's analysis, position cues act as part of a relevant stimulus compound, i.e., "soap-dish - left" and "pot-cover - right." When kind of candy is perfectly correlated with position, distinctive anticipatory responses add differential cues to the compounds, i.e., "marshmallow-soap-dish - left" and "Hershey Kiss-pot-cover - right." The high attention-value of these implicit candy expectations is shown by the superiority of the Successive-D over the Successive-R Group.

The other matched groups were run on a simultaneous junk problem, 16 under the experimental condition of candy-position correlation (Simultaneous -D) and 16 under a control condition with a single kind of candy (Simultaneous -S). Data were also available from 58 comparable subjects under another control condition with random selections of candy (Simultaneous -R). For these simultaneous problems, the correlation of kind of candy with position adds distinct anticipatory cues which are irrelevant, hence detrimental to learning. Significant differences in favor of the two control conditions appear during the first daily block of 25 trials and then tend to disappear, providing some further confirmation of theoretical expectations.

This experiment has been replicated by Shepp using more difficult discriminanda (Color-Form Object) with even clearer results, again in the expected directions. As evidence that rewards act as potent cues and controllers of attention, these findings are clearly reliable, but still largely indirect and inferential. More evidence of the cue-power of rewards comes from another experiment by Shepp in which distinctive candies were used experimentally as both cues and reinforcements.

On one wedge a colored M&M candy was mounted, on the other a miniature marshmallow was encased in clear plastic. The experimenter selected one of these as the positive stimulus, the other as the negative. Under the positive stimulus, a candy (either a marshmallow or M&M) was hidden, and under the negative stimulus, there was, of course, nothing. This task can be classified as a candy-object discrimination with candy reinforcement.

Shepp ran three groups of 13 children (MA 2-5): (a) Group PC in which there was a positive correlation between discriminanda and incentives, that is, the positive stimulus and the incentive were the same kind of candy; (b)

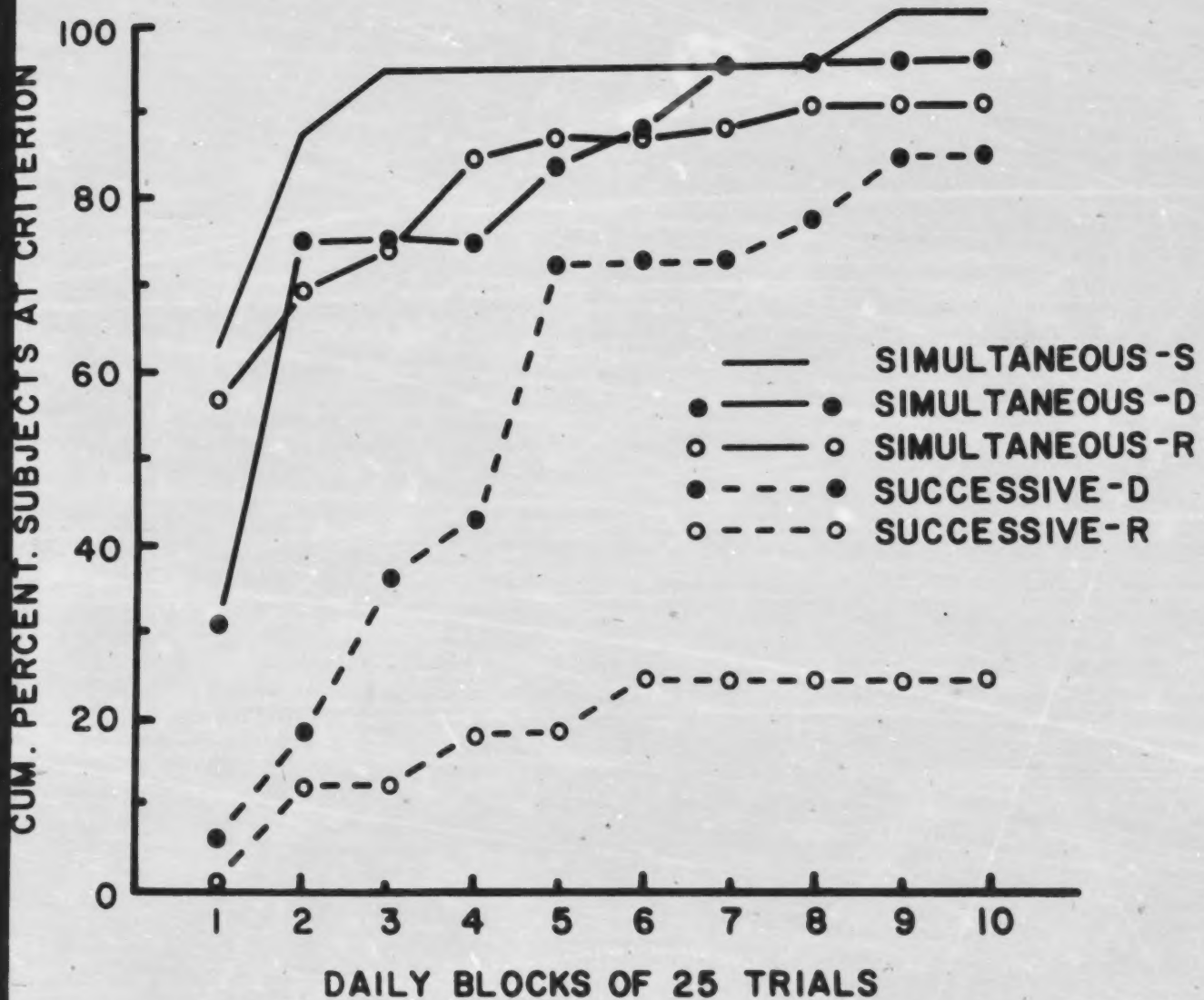


Figure 32. Cumulative percentages of subjects at criterion under five of Shepp's experimental conditions. Differences among the conditions provide evidence for inferring that rewards may serve as strong cues.

Group NC having a negative correlation between discriminanda and incentive, that is, the negative stimulus and the candy used as reward were the same; and (c) Control Group for which the incentive was unlike either the positive or negative stimulus in color or form.

Results of these three groups are shown in Figure 33. It can be seen

Insert Fig. 33 about here

that when the reward is the same as the positive cue, learning is particularly fast. When the reward matches the negative cue, learning is slow. Both comparisons are in relation to the Control Group in which the candy cues bear no relation to the candy reinforcements. For the subject at the moment of choice to make a match between a present candy and an absent one (the incentive) appears to require some mechanism of anticipation which makes incentives a source of cues available in some form prior to choice.

While anticipatory mechanisms (such as S-S connections or anticipatory goal reactions) are extra-theoretical as far as our present model goes, a major consequence of assuming anticipation is that attention theory then applies to the feedback of anticipation. Incentive cues become another stimulus dimension competing for attention with other, more direct, sources of stimulation.

Oddity. - The oddity problem has often been employed to study discriminative processes of primates and children, and it is of especial interest in the context of attention theory, since it can be readily analyzed as requiring both observing and instrumental habits.

To describe a variant of the oddity problem we have used, let A,B,C,D,E, and F represent different stimuli. A typical trial sequence with three simultaneously presented stimuli might be,

Trial 1.	A ⁺	B	B
Trial 2.	C	C	D ⁺
Trial 3.	E ⁺	F	F

with the plus sign indicating the rewarded choice. The odd stimulus is always correct, and since the odd one never appears in the central position, the problem becomes a two-choice discrimination as the subject learns never to select the center stimulus.

The instrumental component of the oddity habit may be specified as "approach the odd stimulus." The exercise of this habit is, of course, contingent upon the subject seeing in what respect the stimuli differ. If the odd stimulus differs from the other two along a single dimension, the subject cannot solve the problem unless he observes the single relevant dimension. The interesting feature of the instrumental oddity habit, according to our analysis, is that once learned, it can transfer to any new situation in which oddity is present. In other words, the oddity habit permits the subject to perform correctly on any oddity problem with no training providing only that the relevant dimension (or dimensions) is observed.

In terms of the theory, $Pr_{(1,0)}$ approaches unity and $Po_{(1,0)}$ is under the control of the particular dimension selected (see Table 2). The usefulness of an oddity habit for attention theory is that it allows us to measure more or less directly the initial attention value ($Po_{(1,0)}$) of new dimensions, uncomplicated by the necessity of teaching a correct instrumental habit ($Pr_{(1,0)}$) at the same time.

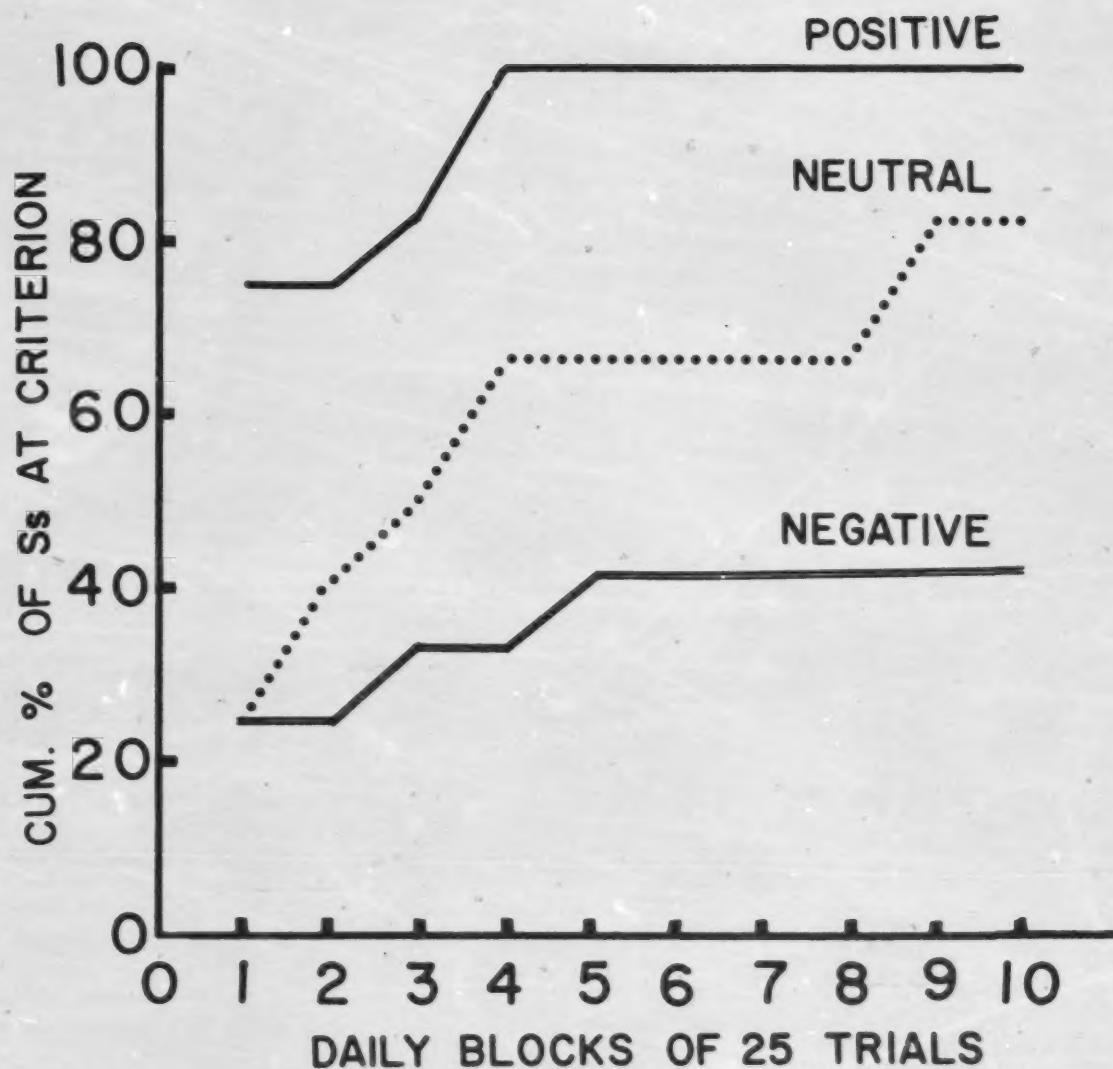


Figure 33. Performances of Shepp's Group PC (labelled "Positive") featuring a positive correlation between discriminanda and incentives, Group NC (labelled "Negative") representing the corresponding negative correlation, and the Control Group (labelled "Neutral").

The logic tying these results to the cue-value of reward is presented in the text.

The major difficulty lies in teaching the oddity habit to low level retardates. When training begins with mixed sets of objects, as illustrated above, with oddity the only cue, only 20% (MA 3-4) are able to learn within 1000 trials. A stimulus reversal method increases percentage learning to 50%. The reversal method consists of presenting ABB (position randomized) until A is chosen six trials in succession, then shifting to BAA until the subject learns to choose B. A and B alternate as the odd, correct cue during a series of reversals until the subject begins to reverse with zero errors, indicating a use of the oddity cue. At this stage, most subjects can perform correctly when mixed sets of objects are introduced. Although the reversal method is the most effective we have tried, percentage learning is still disappointingly small. It is hoped that better ways can be found for training the oddity habit.

Other Stimulus dimensions. - It will be noted that among the postulated stimulus dimensions are some for which the physical basis for sensory reception is not obvious. We do not reject for a priori reasons stimulus aspects such as relationships - same-different, larger than, brighter than, etc. - if these can be shown to follow the laws of the model. Another example of a somewhat subtle dimension is that of temporal duration. Orlando (1960) presented evidence that efficient performance on a fixed-interval operant schedule in retardates required the ability to discriminate temporal intervals. Another stimulus aspect, variability, was postulated to be a cue in a study by House, Orlando, and Zeaman (1957) as subjects showed evidence of learning to approach or avoid the stimulus which changed from trial to trial. These examples serve to point up the variety of stimulus properties which may function as dimensions.

Extensions of the Theory

Partial Reinforcement Submodel

The π parameters. - The theory has so far dealt with only the simplest reward schedule of 100% reinforcement for response to the positive stimulus, 0% reinforcement for response to the negative. Consideration is given now to other schedules.

Let π_1 , and π_2 be random probabilities of reinforcement associated with the two physical stimuli serving as discriminanda. π_1 is associated with the "positive" cue, π_2 with the "negative" cue, where positive means the more frequently reinforced cue. By convention, then $\pi_1 > \pi_2$.

The basic equation relating probability of the correct overt response (choice of the more reinforced cue) to theoretical instrumental and observing probabilities (Equation 1) does not change. Neither do the rules of probability changes as summarized in operator form in Table 1. What does change in this submodel is the probability of application of the various operators. Table 4 is a modification of Table 1 including the new parameters π_1 and π_2 .

Insert Table 4 about here

The expected operator methods we have employed earlier can be used again for purposes of approximation. The expected change in mean probability of a relevant observing response is

$$\Delta \bar{P}_O(1) = (0_0/2) \cdot \bar{P}_O(1) \left[\frac{(\pi_1 - \pi_2)(2\bar{P}_r(1) - 1) - (2\pi_1 - \pi_2)}{\left(n - \sum_{i=2}^n \frac{1}{1 - \bar{P}_O(i)} \right)} \right]$$

... Equation 7

Table 4

Operators Applied to Four Probabilities
 Given the Various Sequences of Events
 That Can Occur on a Trial in Experiments
 With Schedules of Partial Reinforcement

Sequence of Events	Probability of Sequence	Operators Applied To			
		Po(1)	Po(2)	Po(n)	Pr(1)
$O_1 R_1 G$	$Po(1) Pr(1) \pi_1$	Q_A	Q_e	Q_e	Q_A
$O_1 R_1 G'$	$Po(1) Pr(1) (1 - \pi_1)$	Q_E	Q_{a1}	Q_{a1}	Q_E
$O_1 R_1' G$	$Po(1) (1 - Pr(1)) \pi_2$	Q_A	Q_e	Q_e	Q_E
$O_1 R_1' G'$	$Po(1) (1 - Pr(1)) (1 - \pi_2)$	Q_E	Q_{a1}	Q_{a1}	Q_A
$O_2 R_2 G$	$Po(2) Pr(2) (\pi_1 + \pi_2) / 2$	Q_e	Q_A	Q_e	Q_I
$O_2 R_2 G'$	$Po(2) Pr(2) (\pi_1 + \pi_2) / 2$	Q_{a2}	Q_E	Q_{a2}	Q_I
$O_2 R_2' G$	$Po(2) (1 - Pr(2)) (\pi_1 + \pi_2) / 2$	Q_e	Q_A	Q_e	Q_I
$O_2 R_2' G'$	$Po(2) (1 - Pr(2)) (\pi_1 + \pi_2) / 2$	Q_{a2}	Q_E	Q_{a2}	Q_I
$O_n R_n G$	$Po(n) Pr(n) (\pi_1 + \pi_2) / 2$	Q_e	Q_e	Q_A	Q_I
$O_n R_n G'$	$Po(n) Pr(n) (\pi_1 + \pi_2) / 2$	Q_{an}	Q_{an}	Q_E	Q_I
$O_n R_n' G$	$Po(n) (1 - Pr(n)) (\pi_1 + \pi_2) / 2$	Q_e	Q_e	Q_A	Q_I
$O_n R_n' G'$	$Po(n) (1 - Pr(n)) (\pi_1 + \pi_2) / 2$	Q_{an}	Q_{an}	Q_E	Q_I

The corresponding equation for an irrelevant observing response, $\bar{P}o_{(j)}$ is

$$\Delta \bar{P}o_{(1)} = (\theta_o/2) \bar{P}o_{(j)} \left[\bar{P}o_{(1)}/1-\bar{P}o_{(1)} \left\{ (1-2\bar{P}r_{(1)}) (\pi_1 - \pi_2) + (2-\pi_1-\pi_2) - (2-\pi_1 - \pi_2) \left(n - \sum_{\substack{i=2 \\ i \neq j}}^m \frac{1}{1-\bar{P}o_{(i)}} \right) \right\} \right] \quad \dots \text{Equation 8}$$

And for an approximation of the expected change in mean probability of the correct instrumental response, we write

$$\Delta \bar{P}r_{(1)} = \theta_o \bar{P}o_{(1)} \left[\bar{P}r_{(1)} (\pi_1 + \pi_2 - 2) + 1 - \pi_2 \right] \quad \dots \text{Equation 9}$$

Values of $\bar{P}o_{(1)}$ and $\bar{P}r_{(1)}$, computed by means of Equation 7, 8 and 9 can then be substituted into Equation 6 to predict $\bar{P}$, the expected change in mean probability of a correct observable response.

Stat-children under partial reinforcement. - We have not yet computed Stat-children functions for a wide variety of conditions including the schedule parameters π_1 and π_2 . An example of those we have is plotted in Figure 34 with $\pi_1 = .75$ and $\pi_2 = .25$. Values of θ and n are of the order of

Insert Fig. 34 about here

magnitude of those used earlier for regular reinforcement, and starting probabilities are those that might be appropriate for original learning and reversal.

These parameter values turn out to be obviously incorrect for the group experimental data shown in Figure 27, although a few common properties are shared. Chief among these is the fact that both appear to be approaching slowly some intermediate asymptote less than unity. The experimental data for groups are closer in level to a "matching solution" of 75% than the Stat-children groups, but it will be shown in the next section that the apparently stable level of group response during 200 trials with partial reinforcement is under the control of another parameter, the number of relevant dimensions.

Number of Relevant Dimensions and Degree of Relevance: the $\pi \rho$ Model

A new parameter, ρ . - For purposes of simplicity, consideration has been restricted to single relevant dimensions, even though it is quite likely that more than one relevant dimension may be present experimentally. Not only may the number of relevant cues vary, but the degree of relevance of each may also be a variable. To handle these possibilities, we introduce a new parameter, ρ , to represent degree of relevance.

Let ρ_j be the probability that the cues of the j th dimension are associated with (i.e., in the same place as) those of the reference dimension S_1 . As an illustration, suppose S_1 is a form dimension represented experimentally by a square (s_1) as a positive cue, and a circle (s_1') as negative. Also, let S_j be any other dimension (say color) which also splits up into two cues, blue (s_j) and red (s_j').

STAT-CHILDREN

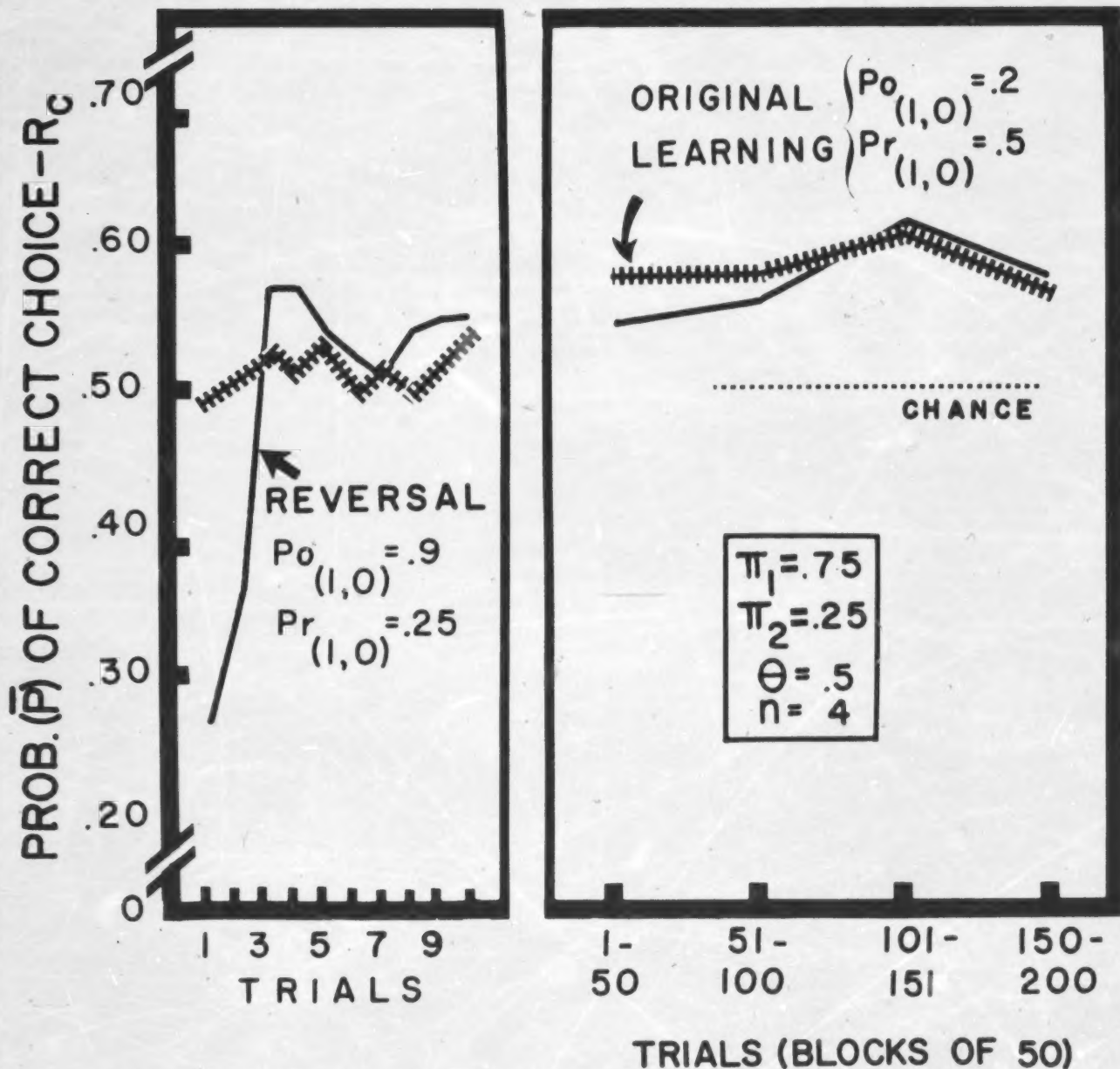
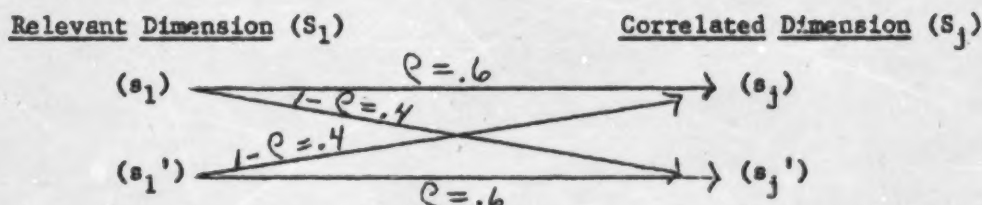


Figure 34. Theoretical learning and reversal curves with schedules of partial reinforcement. Mean probabilities of correct choice of groups of 20 Stat-children are plotted at the left for the first 10 trials of original learning and reversal.

At the right, the same Stat-children are run for 200 trials and their data plotted as means of blocks of 50 trials.

If $P_j = .6$, then the probability that s_1 is associated with s_1 is .6. In terms of the example, the probability that the square (s_1) is also blue (s_j) is .6. It follows from this that the probability of the square being red is .4, as is the probability of the circle being blue.

Restating schematically, the arrows indicate probabilities of association:



To interpret this model experimentally we arrange for these associations to hold.

Any number of dimensions may be correlated with the reference dimension, and for convenience we designate the number of relevant dimensions n_r and the number of irrelevant dimensions n_i . Thus, the sum of n_r and n_i equals n , the total number of dimensions.

Table of operators for the π^P model. - To achieve generality the new parameter is incorporated into the partial reinforcement or π Model to yield the π^P Model. The rules of probability changes for this model are given in Table 5. The operators remain the same as before, but the probabilities

Insert Table 5 about here

of applying them differ, and furthermore, the necessity arises of calculating the probabilities of the instrumental responses to the cues of the correlated dimensions ($Pr_{(j)}$), since these no longer remain at chance.

Basic equation. - Unlike the π model, the π^P model requires a new basic equation for computation of P .

$$P = Po_{(1)}Pr_{(1)} + \sum_{i=2}^n Po_{(i)}Pr_{(i)}P_{(i)} + \sum_{i=2}^m Po_{(i)}(1-Pr_{(i)})(1-P_{(i)})$$

. . . Equation 10

A verbal translation of this equation says: the correct (reference) stimulus will be lifted if the subject looks at the correct dimension and approaches the correct cue, or he looks at one of the nonreference dimensions and approaches one of the cues (s_1) which is associated with the correct stimulus, or he looks at one of the nonreference dimensions and approaches the other cue (s_1') which is associated with the correct stimulus.

The reference dimension is any dimension perfectly correlated with reinforcement, and all the others are nonreference dimensions. The nonreference dimensions are relevant if their P values are 1.0, irrelevant if their P values are .5, and partially relevant or correlated if their P values lie between .5 and 1.0.

Equation 10 can be simplified to

$$P = Po_{(1)}Pr_{(1)} + \sum_{i=2}^m Po_{(i)} \left[Pr_{(i)} (2P_{(i)} - 1) + (1 - P_{(i)}) \right]$$

. . . Equation 11

which reduces to Equation 1 when all $P_{(i)}$ are equal to .5.

Table 5

Operators and Probabilities of Application for the πe Model

Sequence of Events	Probability of Sequence	Operators Applied To				
		Po ₍₁₎	Po _(j)	Po _(n)	Pr ₍₁₎	Pr _(j)
O ₁ R ₁ G	Po ₍₁₎ Pr ₍₁₎ π_1	Q _A	Q _e	Q _e	Q _A	Q _I
O ₁ R ₁ G'	Po ₍₁₎ Pr ₍₁₎ (1- π_1)	Q _E	Q _{al}	Q _{al}	Q _E	Q _I
O ₁ R ₁ 'G	Po ₍₁₎ (1-Pr ₍₁₎) π_2	Q _A	Q _e	Q _e	Q _E	Q _I
O ₁ R ₁ 'G'	Po ₍₁₎ (1-Pr ₍₁₎)(1- π_2)	Q _E	Q _{al}	Q _{al}	Q _A	Q _I
O _j R _j G	Po _(j) Pr _(j) $[e_j \pi_1 + (1-e_j) \pi_2]$	Q _e	Q _A	Q _e	Q _I	Q _A
O _j R _j G'	Po _(j) Pr _(j) $[1 - e_j \pi_1 - (1-e_j) \pi_2]$	Q _{aj}	Q _E	Q _{aj}	Q _I	Q _E
O _j R _j 'G	Po _(j) (1-Pr _(j)) $[e_j \pi_2 + (1-e_j) \pi_1]$	Q _e	Q _A	Q _e	Q _I	Q _E
O _j R _j 'G'	Po _(j) (1-Pr _(j)) $[1 - e_j \pi_2 - (1-e_j) \pi_1]$	Q _{aj}	Q _E	Q _{aj}	Q _I	Q _A
O _n R _n G	Po _(n) Pr _(n) ($\pi_1 + \pi_2$)/2	Q _e	Q _e	Q _A	Q _I	Q _I
O _n R _n G'	Po _(n) Pr _(n) ($\pi_1 + \pi_2$)/2	Q _{an}	Q _{an}	Q _E	Q _I	Q _I
O _n R _n 'G	Po _(n) (1-Pr _(n))($\pi_1 + \pi_2$)/2	Q _e	Q _e	Q _A	Q _I	Q _I
O _n R _n 'G'	Po _(n) (1-Pr _(n))($\pi_1 + \pi_2$)/2	Q _{an}	Q _{an}	Q _E	Q _I	Q _I

Stat-children functions. - Equation 11 is used in conjunction with the table of operators to compute Stat-children functions. Figure 35 shows the effects of variation in number of relevant dimensions (n_r) for three groups

Insert Fig. 35 about here

of 20 Stat-children. It is evident from these functions that increasing the number of relevant dimensions raises the level of group performance. The theory agrees in this respect with data presented earlier (Figure 6) showing that two cues (color and form) resulted in greater learning than either alone. The relative ease of multidimensional (junk) stimulus learning is another empirical finding consistent with this property of the π Model.

Inclusion of the π parameter results in the functions plotted in Figure 36. The effects of schedules is seen to be under the control of the number of relevant dimensions. Increasing n_r from 1 to 4 elevates the level of performance to about 75% for the group function, within 200 trials. The empirical data on position discrimination presented earlier (Figure 27) is more similar to the 4-relevant dimension Stat-child function than the curve for one relevant dimension. The implication is that position may be multidimensional in nature - a not unintuitive proposition.

If we increase the ρ value of a correlated dimension from .50 to .75, it seems to have no marked effect detectable with means of 20 Stat-children with the parameters used in Figure 36, while increasing ρ value from .75 to 1.0 (and thus making two relevant dimensions) has a marked effect. We have as yet no retardate data to test this interesting and easily testable prediction. But corroborative evidence has been found with rats by Jeeves and North (1956) and by Babb (1956). Both studies showed that increasing correlation between irrelevant and relevant dimensions from .50 to .70 or .75 did not produce any differences in learning speed.

One important question is whether or not a distinction should be made between irrelevant stimuli which vary and those which remain constant. For example, if form were relevant, variable colors, such as red and green, might be paired at random with the correct or incorrect form cues. Conversely, the color of the correct and incorrect form cues might be the same, always red for example, for a constant irrelevant cue. Theoretically, no distinction is made between these two conditions; in both cases ρ is presumed to be equal to .5. Extra-theoretical considerations, however, might suggest that variable cues have more attention value, in which case we would expect variable irrelevant cues to retard learning in comparison with constant irrelevant cues. Evidence is available from rat studies indicating that variability may or may not retard learning depending upon the relative strength of relevant and irrelevant dimensions. Three studies have compared variable black-white irrelevant cues with constant brightness as an irrelevant dimension. Variability did not interfere when bars-no bars was the relevant dimension (Jeeves and North (1956)), but did retard learning when chains-no chains (Babb, 1956) or upright-inverted triangles (Wortz and Bitterman, 1953) were relevant. Elam and Bitterman (1953) found that learning a thick-thin stripe discrimination was not hindered by variable presentation of a horizontal or vertical display. But when this training was followed by an extradimensional shift condition with horizontal-vertical stripes relevant, variability along the previously trained thick-thin dimension was detrimental in comparison with a constant condition.

Expected operator functions. - The expected operator equations have been worked out for this model as before, but the results are cumbersome in form. They are omitted for space considerations.

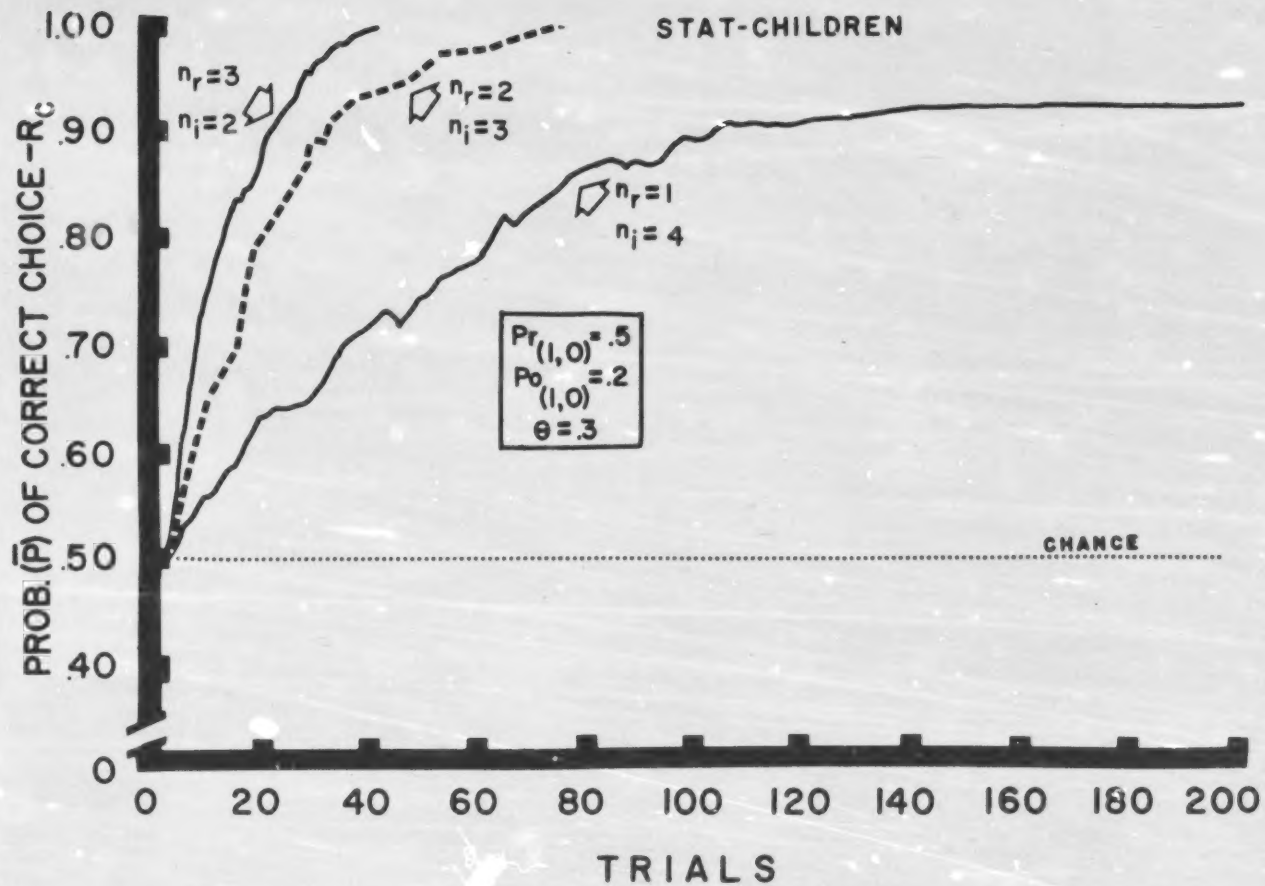


Figure 35. Theoretical effects of increasing the number of relevant dimensions. Average performance levels of three groups of 20 Stat-children improve as the number of relevant dimensions increases from one to three.

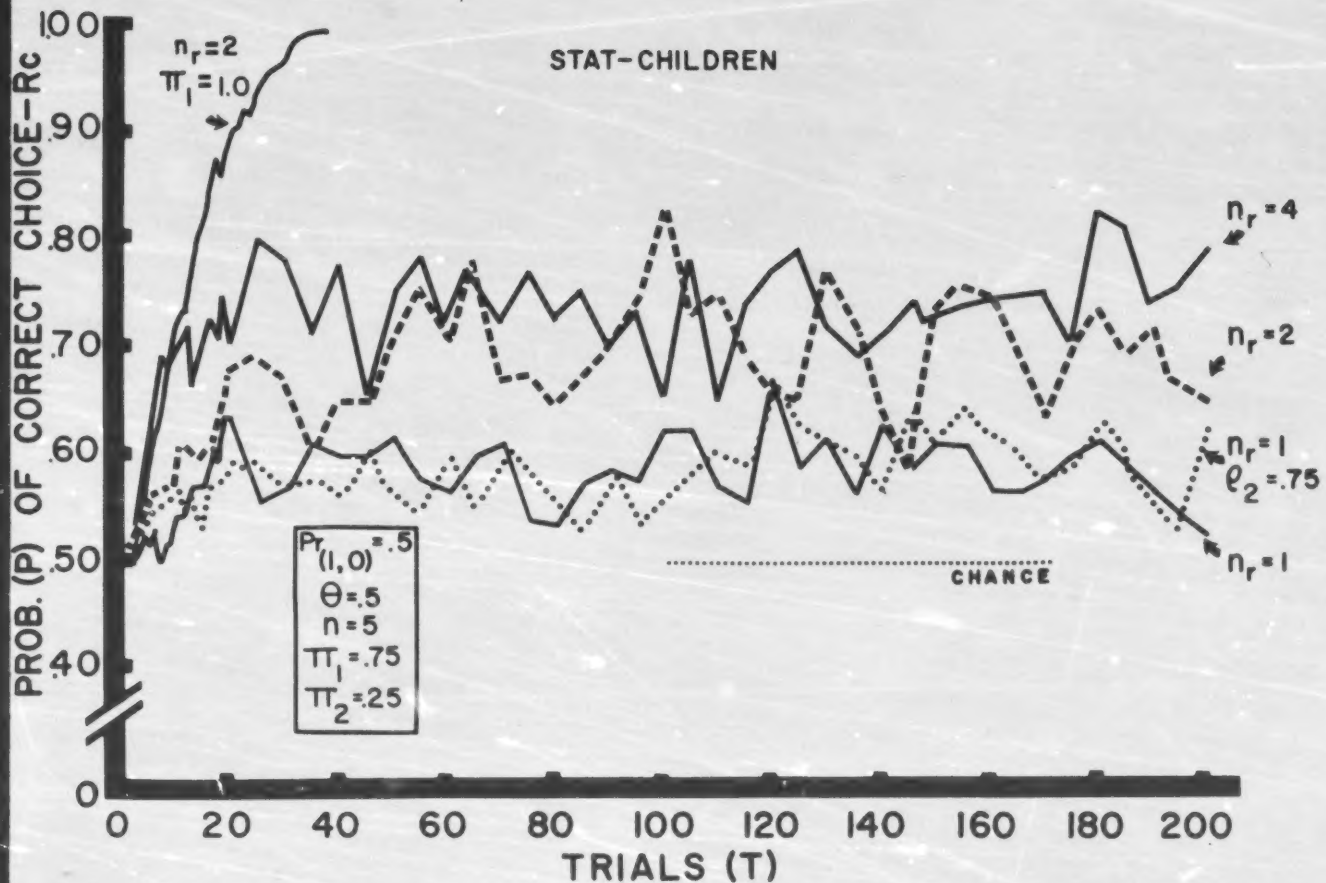


Figure 36. The joint effects of schedules of reward and number of relevant dimensions on Stat-child performance. All groups were run on a 75-25 schedule with the exception of the fastest learning group which had a 100-0 schedule. Stat-children represented by the dotted function had one relevant and another partially relevant ($\rho_2 = .75$).

Two-look Model

Without abandoning our central theoretical notion that attention is a limited capacity, we may try to relax our assumption that the subject looks at only a single dimension at the moment of choice. Let us start with the new assumption that a maximum of two dimensions may be simultaneously attended to at the moment of choice. To simplify further, we restrict the number of dimensions considered to just two and let $\pi_1 = 1.0$ and $\pi_2 = 0$.

Figure 37 paradigm the possibilities on any trial. The subject observes

Insert Fig. 37 about here

either one of the two dimensions, or both ($0_{(1+2)}$), in which case four possibilities arise. There is the probability, P , that the two cue compounds s_1s_2 (on one side) and $s_1's_2'$ (on the other) will result from $0_{(1+2)}$, and there is the complementary probability ($1 - P$) that the other two cue compounds will be made available. Associated with each of the four cue compounds must be an instrumental approach response with conditional probabilities as diagrammed.

The basic equation for such a model is

$$P = P_{0(1)}Pr_{(1)} + P_{0(2)}Pr_{(2)}P_2 + P_{0(2)}(1 - Pr_{(2)})(1 - P_2) \\ + P_{0(1+2)}Pr_{(1+2)}P_2 + P_{0(1+2)}Pr_{(1+2)}(1 - P_2)$$

Operator tables and expected operator functions are not sufficiently different from the other models to warrant presentation. Generalization to an N-look model offers no particular difficulty other than a great deal of book-keeping if we accept as our goal the generation of Stat-children functions. We have as yet no Stat-children run on this model.

General Discussion

Some Limitations of the Theory

The theory is not intended to give a complete account of discrimination learning. Rather it is an attempt to derive consequences of a few basic assumptions. Re-stated qualitatively, these are: (a) attention is limited to only one (or, at most a few) of the many possible stimulus dimensions available to the subject at the moment of choice; (b) subjects may learn to attend to or disregard stimulus aspects as a result of differential reinforcement; and (c) cues for instrumental learning are those aspects of the stimulus which are being attended to. We have already given a detailed account of many of the empirical effects these assumptions can handle. We now turn to a brief discussion of some of the phenomena which the present formulation is not equipped to handle.

New perceptual learning. - The theory is not one of perceptual learning in the sense of acquisition of new perceptual responses. Each subject is assumed to have a set of dimensions with varying probabilities of being observed. But no description is given of how the subject gets these dimensions to begin with, or how the number available changes with age. Thus, the theory is to be distinguished from those which have as a goal the explanation of perceptual development, such as Hebb's neurophysiological account (1949) of how organisms learn to perceive or Werner's (1948) hypothetical levels of perceptual organization. Such theories attempt to account for the level of perceptual development which the present model takes as given. The view of perceptual learning most congenial to the observing response model is that of Gibson and

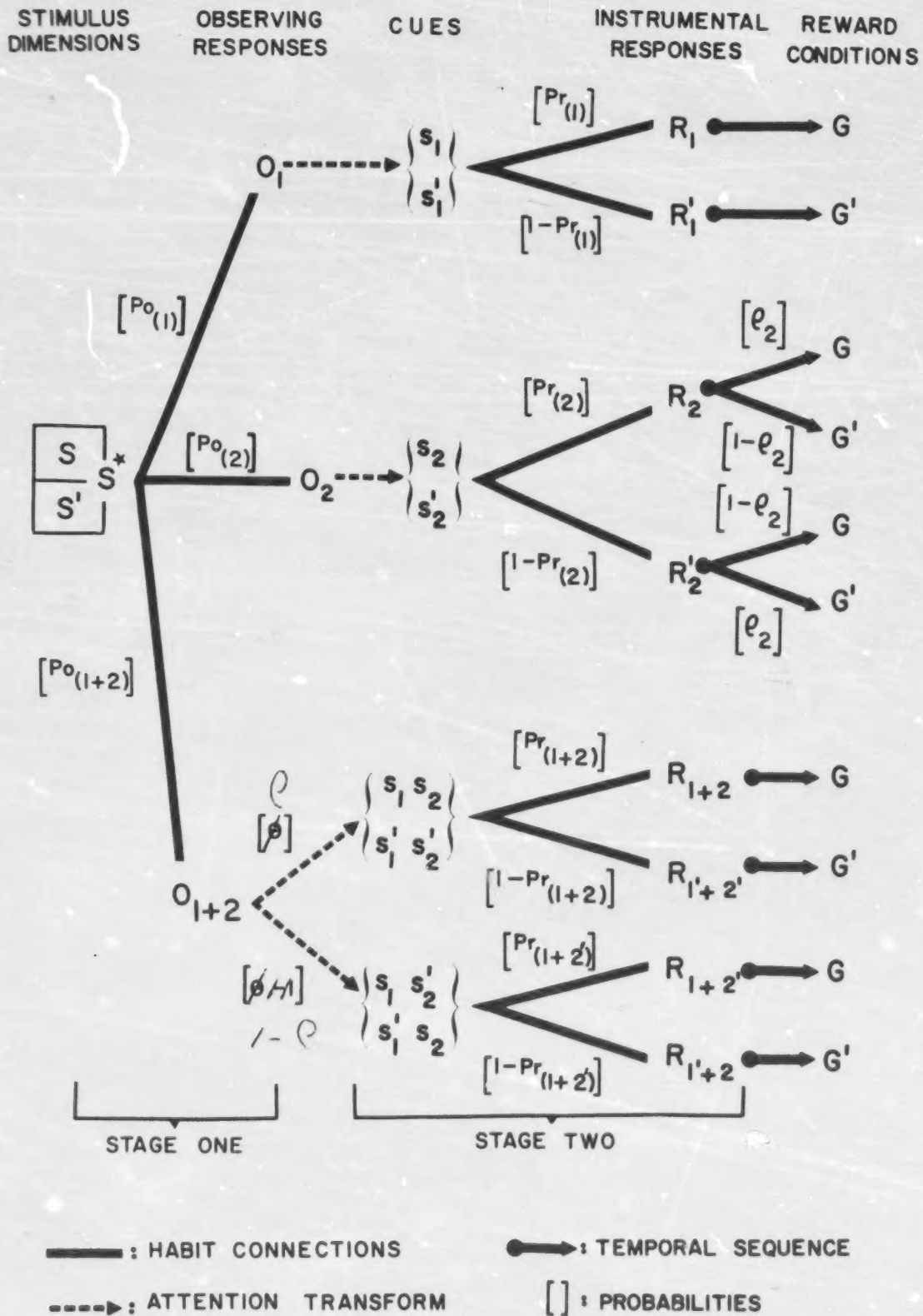


Figure 37. Probability tree for the "Two-look" model.

Gibson (1955; Gibson, 1959). They emphasize the wealth of information in the complexity of energies and patterns of energies stimulating the sense organs. Perceptual learning is regarded as a process of discovering new properties of the world by discriminating new variables in the stimulus flux. Perceptual development, then, becomes a matter of differentiating sensory input, rather than a process of reorganizing or reinterpreting sensory information. The notion that organisms learn to discriminate new variables may be roughly equivalent to our supposition that more intelligent subjects observe a greater number of dimensions.

Generalization.- Our theory says nothing specifically about stimulus generalization, and experimentally, we have minimized the problem of generalization by using as discriminanda stimulus properties which if seen are easily distinguished by our subjects. Squares vs. circles, red vs. green, and left vs. right are examples. It is obviously not necessary to experiment in this way. Cues closer together on some continuum might be chosen, and then we would expect the traditional finding that speed of discrimination learning is inversely related to the stimulus distance between discriminanda. Many theories of discrimination (e.g., those of Hull (1943), Spence (1937), Burke and Estes (1957), and Bush and Mosteller (1951) can deduce this finding by postulating gradients of stimulus generalization and rules for their interaction.

Such gradients have been demonstrated with retardates. Smith (1960), for instance, trained a lever-pulling response in a go-no go free operant situation using a light of a given wavelength as the positive stimulus versus no light as the negative stimulus. During extinction, the wavelength of the light was varied. After an initial tendency to respond equally strongly to the test wavelengths (which differed from the training wavelength), the retardates showed a generalization gradient by distributing their responses according to a decreasing function of distance from the training stimulus. The function obtained bore a relation to the normal human discriminability function.

Information on stimulus generalization gradients from such experiments may be used to write additional postulates for attention theory to handle generalization effects. This is left as an important line of future theoretical development. Whatever the new generalization postulates, one consequence can already be predicted: changes in cues of dimensions observed with probabilities close to zero will lead to flat generalization gradients and undisturbed discriminations.

Relationship to Other Theories

Continuity-noncontinuity theories. - Some properties of our model bear on the continuity-noncontinuity controversy without corresponding closely to either position. According to Krechevsky's (1938) version of noncontinuity theory, the organism selects a certain set of discriminanda to which he reacts, i.e., entertains a Hypothesis. While paying attention to some particular set of stimuli, the animal learns only about that set. If he is attending to some irrelevant set of stimuli, he eventually gives up the "Hypothesis" and adopts another. He learns nothing about the relevant cues until such time as he has adopted the relevant Hypothesis. This account has some similarity to our "one-look" observing response (OR) model in that the subject learns only about dimensions he is attending to. One difference between OR and Hypothesis is that a Hypothesis is assumed to hold over a block of trials, while OR theory postulates that over a block of trials several dimensions may be observed, one per trial, corresponding to their probabilities of being observed. Another difference is that ORs are presumed to follow the same laws, such as acquisition and extinction, as any other response, while Hypotheses are cognitive states, not necessarily bound by the laws of Habit.

Spence (1945), a continuity theorist, described discrimination learning as the gradual growth of differential excitatory tendencies to the positive and negative cues resulting from rewards and nonrewards. He assumed that excitatory strength was added to every stimulus property which impinged on the sense organs of the animal in close temporal contiguity with response and reward. His theory was intended to apply, however, when the organism was certain to be exposed to the relevant cues on every trial. He made the qualification that in many discrimination situations, performance might be influenced by the necessity for the animal to learn to orient toward relevant aspects of the surround. Orienting responses and ORs follow similar laws of acquisition and extinction, but they differ in degree of selectivity. The orienting response is defined peripherally and has the function of merely insuring that receptors are stimulated by the relevant stimulus. The OR makes a further selection among the stimulus dimensions impinging upon the sensorium, thereby limiting instrumental learning to a single dimension. Results of experiments designed to test Spence's theory versus Krechevsky's Hypothesis theory are not critical for the OR model, which can predict results compatible with either position, depending upon experimental conditions.

Acquired distinctiveness and similarity of cues. - The observing response model may also be seen to belong to a class of theories employing the concepts of acquired distinctiveness and similarity of cues. There is considerable evidence that the positive and negative transfer effects from which these concepts are inferred are present in rats (Lawrence, 1949, 1950) as well as humans (Goss, 1955, 1961). Most of the theories which have been advanced to account for these transfer effects have consisted of extensions or modifications of the continuity position, in the form of inferred mediating responses elicited by the discriminative stimuli. Two types of mediating responses have been postulated - those which act by adding distinctive or similar cues to those already present (e.g., verbal labels) and those which exercise a selective effect on the environment (e.g., orienting response, observing response.)

Response-produced cues. - Many of the effects considered by the present model have been attributed to response-produced cues, i.e., those added to the stimulus situation by mediating responses. Goss (1955, 1961) has reviewed these theories and the experimental evidence. Kendler and Kendler (1959, 1960) have incorporated verbal mediation into a theory of child development. They maintain that Spencean, one-stage, discrimination theory is adequate to account for all behavior of non-verbal or pre-verbal organisms. They present evidence that the superiority of reversal over extradimensional shift occurs only in older children (5-7 years) who learned the original problem quickly, but not in younger subjects (3-5 years). The improvement in reversal performance with age is attributed to the process of language development and the increasing control of verbal mediating responses on behavior.

Observing response theory does not exclude response-produced cues as stimulus variables affecting behavior. As a part of the total sensory input, these stimuli may be observed or not observed in the same manner as those arising from external stimulation. We have made no assumptions that response-produced cues are particularly potent as compared with other cues. Verbal mediating responses are of obvious importance in accounting for human behavior, not only as a device for adding cues but as a method for more direct control of behavior by means of complex responses attached to verbal cues during previous training history. For the experimental data presented in this chapter, verbal mediation as a factor cannot be ruled out. A discussion of whether or not all the data presented can be accounted for by Spence's theory plus the assumption of verbal mediation is not within the scope of this paper.

Selective mechanisms. - Among theorists who have posited that organisms select particular aspects of the stimulus situation to respond to, while ignoring others are Reid (1953) and Lawrence (1949, 1950; Goodwin & Lawrence, 1955). Reid proposed that as animals learn to make the correct response they are also learning a response of "discriminating," i.e., "learning to respond to a set of stimuli of which the specific stimulus is a member." As evidence, Reid analyzed the performance of rats on a discrimination reversal problem, showing that overlearning trials on the original problem increased speed of reversal.

Goodwin and Lawrence (1955) postulated a dual process - one of identification or reaction to a dimension and the other that of instrumental learning. They assumed that the identification process can be learned and extinguished. Evidence to support their view came from a study in which rats were given a series of extradimensional shifts, black-white and high-low hurdles alternating as relevant cues. The rats were able to retain an old habit (e.g., black-white) while being trained on a new discrimination (e.g., high-low hurdles) in spite of the fact that during hurdle training, black and white were being randomly reinforced. Goodwin and Lawrence inferred that the rats had quickly extinguished the identification response to the black-white dimension, so that the random rewards and nonrewards were ineffective. The properties of the identification response are quite similar to those of the observing response.

Some other writers who have postulated selective mechanisms can be classified as probability theorists and are discussed in the next section.

Wyckoff. - Our models are based on ideas similar to those of a number of theorists, but especially Wyckoff. Wyckoff (1952) defines the "observing response" as any response which results in exposure to the relevant cues, and relates discriminative performance to the product of two probabilities, that of making the observing response, and of making the correct instrumental response (Cf. Equation 1). He discusses the general properties of any theory in which the frequency of effective instrumental training trials depends upon the probability of observing the relevant cue, and in which (to complete a circular dependency) the increase in probability of observing is a function of the strength of the instrumental habit.

In a later paper, Wyckoff (1954) adds the assumption to his model that the growth of the observing response is a function of the secondary reinforcing power of the discriminative stimuli. An electronic model was constructed which operated according to the postulates of the system to generate learning curves for original learning, reversal, and extradimensional shift.

While, in general, Wyckoff's model has many similar properties to the present theory, a number of points of divergence should be noted. (a) Wyckoff's specification of the observing response differs from ours in not being dimensional in number. (b) The mechanism for strengthening the observing response is secondary reinforcement in Wyckoff's model, while we assume that the same rewards and nonrewards act on both instrumental and observing responses. (c) We use different operators than Wyckoff as rules of probability changes. (d) The various extensions of our basic model incorporating the parameters π and Q and two-looks are not Wyckoffian.

Stochastic models. - From Estes (1959) and Bush and Mosteller (1955) we have borrowed our operators, some rules of notation, and methods of getting approximations. The discrimination models of Burke and Estes (1957) and Bush and Mosteller (1951) are, however, quite different from ours in that these theorists do not conceive of discrimination as requiring a chain of two responses.

Atkinson and Restle. - An "observing response" model based on different assumptions has been developed by Atkinson (1958). The organism is assumed to make either an "observing response," in which case he sees only the relevant stimulus, or a "non-observing response" which exposes him to both the relevant and irrelevant cues. The relevant cues, therefore, begin to be conditioned from the beginning of training, and as trials increase, attention is narrowed so that only relevant cues are sampled. Restle's (1955) discrimination theory

achieves somewhat the same effect by assuming that as learning progresses, relevant cues are sampled on every trial and gradually become conditioned. Irrelevant cues, on the other hand, become adapted so that they no longer have any effect on behavior. Bourne and Restle (1959) applied this theory to the problem of the effect on performance of varying numbers of relevant and irrelevant dimensions. Their predicted effects were in the same direction as those predicted by the present model. However, for purposes of describing retardate learning, both Atkinson's and Restle's models have the disadvantage of assuming that learning of the instrumental response begins on the very first trial. Our data suggest strongly that an adequate model must allow for the possibility that instrumental learning is completely absent under some circumstances for a number of trials.

Attention Theory and Retardation

A major goal in the development of this attention model is to provide a framework for analysis of apparent learning deficits in retardates. In this section we discuss the relationships between parameters of the model and intelligence.

Invariance of θ . - One conclusion drawn was that learning rate (θ) is not a particularly important source of variance in the discrimination learning of retardates. We were led to this conclusion by examination of the slopes of the backward learning curves - the most sensitive index of rate differences. With MAs ranging from two to eight years, the final portions of backward curves were not distinguishably different. We have further shown that even in "slow learners", abrupt mastery of problems may be obtained by changing stimulus aspects or training procedures. Ruling out learning rate as a factor varying with intelligence may appear to violate intuitive assumptions about the nature of intelligence, since standard definitions have long included the ability to learn as a criterion. However, this conclusion is not unprecedented. Both Woodrow (1946) and McPherson (1948, 1958) have stressed the predominately negative results which have arisen from attempts to demonstrate a relationship between learning rate and intelligence.

Role of P_o (1.0). - A low P_o for the relevant dimensions, we have argued, is a major cause of the retardates' poor performance. That is, the color-form dimensions used in our experiments happened to have low probabilities of being observed by subjects of low mental age. But a low P_o for color or form must mean (other factors constant) a correspondingly higher probability of observing some other ("easier") dimension, since all P_o s must sum to unity. A curious consequence of this state of affairs is the prediction that for certain "easy" dimensions, lower MA children would learn to discriminate faster than higher MA children. This odd prediction is not without some experimental support. Weir and Stevenson (1959) report that nine-year old normal subjects did significantly poorer than five-year olds on a simple discrimination task using animal pictures as stimuli.

The n parameter. - Comments of the older subjects in the Weir and Stevenson experiment indicated that they were looking for something more complicated than the simple solution. Theoretically, this might be interpreted as reflecting individual differences in the n parameter. Older and brighter subjects see more dimensions. While a large n parameter could conceivably mean faster learning if the experimenter uses many simultaneously relevant dimensions, it usually implies that the subject will see more irrelevant dimensions and learn more slowly because of the common practice of using a limited number of relevant dimensions (i.e., "hard" problems). This implication of poorer performance by brighter subjects appears paradoxical in view of the positive correlation we find between intelligence and ability to solve color-form discrimination problems.

How can we assume that subjects of higher developmental level (a) see more (irrelevant) dimensions, (b) have, therefore, more irrelevant observing responses to extinguish than less intelligent subjects, (c) learn and extinguish (in the sense of 9) at the same rate as lower level subjects, and then predict superior performance for the brighter subjects?

A possible resolution of this paradox may be derived from the theoretical fact that the relative strengths of irrelevant observing responses is also a determinant of performance (See Figure 14). A large number of irrelevant dimensions may not retard discrimination appreciably if a few of the irrelevant dimensions have high initial Pos, while the others are low. We speculate that intelligent subjects may have learned for specific stimulus situations (S*s) to restrict attention largely to those aspects which have paid off in the past. The less intelligent subjects are regarded as distributing attention more evenly over their (fewer) observable dimensions.

We contrast children who are capable of discriminating many aspects of the situation, but ignore (relatively) all but those likely to be associated with reinforcement, with children who are able to distinguish only a few aspects but attend to all about equally. The less intelligent subjects might earn the label "distractible" even though they have fewer dimensions to distract them. The answer to the paradox, we believe, lies in the distribution of probabilities of distracting dimensions (Po₍₁₎) not simply in the number (n₁) of irrelevant dimensions.

Some Implications for Training

If our analysis of retardation and attention is correct, the secret of successful training of moderately retarded children lies in the engineering of their attention. In training tasks which require discrimination, one should seek ways of increasing the attention value of the relevant cues. Such methods may include manipulation of the stimulus situation so as to increase noticeability of the relevant dimensions, or the use of transfer operations, such as the object-to-pattern sequence described earlier.

It is of interest to note that some traditional methods for training retarded children are derivable from the present theory. Montessori (1912) is a case in point. While her theoretical principles may be questionable, it is presumed that her techniques were adopted because they were found to work. Montessori advocated the use of three-dimensional cutouts in teaching form discriminations, such as letters and numbers. In explicating her principles of brevity, simplicity, and objectivity, she cautioned teachers not to present or draw attention to irrelevant details, and described ways of making the relevant aspects of the material as prominent as possible. She recommended an easy-to-hard sequence of training in saying that "one should proceed from few stimuli strongly contrasting, to many stimuli in gradual differentiation always more fine and imperceptible." (1912, p.184)

Strauss and Lehtinen (1947) emphasize the elimination of distracting details in describing their techniques for training brain-injured children. They suggest covering windows or having students sit facing a blank wall. Reading material is often enclosed within a cover which exposes only a small area at a time. Such techniques might, in terms of attention theory, serve to reduce the n parameter. Many of the training methods described by Strauss and Lehtinen can be analyzed as ways of directing attention to relevant cues. In counting, for example, the child may be required to place each object on a peg as he gives the number. This method not only maintains attention but produces more distinct feedback cues from the counting act. The work of Fernald (1943) in remedial techniques abounds in examples of ways of controlling attention. Her procedure of requiring the child to trace the letter as he repeats it aloud ensures that the relevant form dimension is being attended to as the verbal response is learned. The easy-to-hard sequence of beginning with large letters

for early training before attempting to teach letters of ordinary printed size is also compatible with the principles of the present theory.

Thus, we see that successful teachers have made use both of stimulus arrangements which increase attention value of relevant cues and of transfer designs which progress from stronger to weaker cues.

A few stimulus factors in controlling attention have been indicated by the present research. We have shown that three-dimensional displays make later pattern learning easier. Novelty is another attention-getting aspect. Recent work has pointed to yet another important factor - the spatial contiguity of cue and incentive. Marked facilitation of pattern learning was obtained in trainable retardates when the pattern was repeated on the inside of the candy container, so that the candy was found directly on top of an exact copy of the discriminative color-form stimulus. Use of this method reduced failure rate from 100% (for subjects of MA 2-4 using the standard method of pattern presentation) to zero. Similar results have been obtained by Jarvik (1956) with chimpanzees. One possible explanation is that spatial contiguity of the candy and pattern increases the probability that the subject will observe the pattern. Or it may be that the close temporal contiguity of observing the pattern and obtaining the reward is the facilitating factor. Whatever the explanation, the effect is a strong one, deserving of further exploration.

We have discussed earlier some specific transfer effects, such as intra-dimensional shift. There is also a kind of generalized transfer termed "learning set" by Harlow (1959), which is characterized by gradual improvement in discrimination performance over a long series of different problems. Learning set acquisition has been shown to vary with intelligence by Kaufman and Peterson (1958), Ellis (1958), and Girardeau (1959). In a typical experiment, the stimuli differ multi-dimensionally, and the same dimensions are not necessarily relevant from one problem to the next. In terms of the attention model, learning set could result from extinction of observing responses to the class of dimensions which are never relevant and the acquisition of strong tendencies to observe those dimensions which are frequently relevant.

A contrasting kind of general interproblem transfer has been observed, which we have called "failure set." (Zeaman and House, 1962b) Trainable subjects suffering prolonged failure on a problem which proved to be insoluble (conditional reaction) were found then to be unable to solve the simplest (junk) problem, although they had previously been able to do so. Failure set receives a ready (although as yet untested) theoretical interpretation as due to extinction of the observing response to broad classes of visual stimuli, such as those associated with the experimental situation. In the absence of any differential reinforcement for observing particular dimensions, the Pos for all dimensions should approach equality. Subjects have not only lowered Pos for the relevant dimension but equal ratios of strength of irrelevant dimensions, a particularly unfavorable condition. Whether or not this explanation is correct, the occurrence of failure sets is taken as a strong argument for adoption of easy-to-hard sequences. If we compare the effect of extradimensional shift with that of prolonged failure, the implication is that even learning the wrong things may be preferable to learning nothing. Even though extradimensional shift gives negative transfer as compared with intradimensional shift, such training is not as damaging as prolonged failure.

Concluding Remarks

A theory has been developed in this chapter that the visual discrimination learning of moderately retarded children requires the acquisition of a chain of two responses: (a) attending to the relevant stimulus dimension, and (b) approaching the correct cue of that dimension. The difficulty that retardates have in discrimination learning is related to limitation in the first, or attention, phase of this dual process rather than the second.

A small collection of stochastic models organized by these general notions has been applied to a series of discrimination experiments with lower level retardates including original learning, reversals, the effects of intelligence, stimulus factors, schedules, and transfer operations.

While the domain of empirical relevance of the theory is given in part by a description of the experimental procedures we have used in studying two-choice visual discriminations, we know of no reasons why the theory might not be found to hold for retardate learning of greater complexity and in other sensory modalities. Nor is it unlikely that the theory may hold for nonretarded subjects of comparable developmental levels.

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